SEQUENTIAL LOCAL TRANSFORM ALGORITHMS FOR GRAY-LEVEL DISTANCE TRANSFORMS

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ABSTRACT

In this paper, new algorithms are presented for the calculation of gray-level distance transforms, in which the distance values of pixels are proportional to the gray-value differences of minimal paths [5], not the gray values themselves, which is the case in [1] and [3]. The presented algorithms are sequential local transform algorithms. The performance of the algorithms are evaluated by testing how quickly they converge to the error-free distance image. It is shown that the presented algorithms are faster than the previously presented algorithms [5] for gray-level distance transforms. It turns out that the 4-neighbor, 4-raster algorithm is the fastest, converging to a distance image with no erroneous pixels in only two iteration rounds. Furthermore, because of the raster scanning approach they are easily applied other image grids than the rectangular one, and are easy to implement.

1 INTRODUCTION

The previously presented distance transforms for gray-level images in [3] and [1] find the minimum path joining two points by the smallest sum of gray levels or weighting the distance values directly by the gray levels. In those methods, the distance values do not depend on gray value differences, but the values themselves.

Fall-distance [4], [7] is another modification of distance, in which the only permitted paths from the reference set are those with falling, i.e. strictly decreasing, gray values. The set of points reached by such strictly decreasing paths is known as the fall-set of the reference set. In [4] a sequential two-pass algorithm is used to calculate the fall-distance.

Recently, a new distance transform for gray-level images has been presented [5], [6]. In this transform, the Distance Transform on Curved Space (DTOCS), the distance values of pixels are proportional to the gray-value differences of minimal paths, not the gray values themselves. This paper presents new and more efficient algorithms for the calculation of the DTOCS.

2 DEFINITION OF THE DTOCS

Definition 1. Let $X \subset Z^2$. Let $B \subset Z^2$ be the structuring element. Let the external boundary of X be denoted by ∂X and be defined by $\partial X = (X \oplus B) \setminus X$. $\partial X \subset X^C$. (Giardina and Dougherty, 1988)

In the definition below we will use the following notation. $x \in X$ and $y \in \partial X$. Let $\Psi_X(x,y)$ denote the set of digital 8-paths in $X \cup \partial X$ linking x and y. Let $\gamma \in \Psi_X(x,y)$ and let γ have n pixels. Let $a_i \in \gamma$ and $a_{i+1} \in \gamma$ be two adjacent pixels in the path γ. Let $\mathcal{G}_X(a_i)$ denote the gray value of the pixel a_i.

The Distance Transform on Curved Space (DTOCS) image defined as follows.

Definition 2. Let the distance between a_i and a_{i+1} be $d_X(a_i, a_{i+1}) = |\mathcal{G}_X(a_i) - \mathcal{G}_X(a_{i+1})| + 1$, $i = 1, 2, ..., n-1$. The length of the path γ is defined by $\Lambda(\gamma) = \sum_{i=1}^{n-1} d_X(a_i, a_{i+1})$. The DTOCS distance image is defined by

$$\mathcal{F}(x) = min(\Lambda(\gamma), \gamma \in \Psi_X(x,y)), \quad (1)$$

$$\mathcal{F}(y) = 0 \quad (2)$$

3 DEFINITION OF THE WDTOCS

The definition of the Weighted Distance Transform on Curved Space (WDTOCS) consists of the Definition 1 and the following Definition 3:

Definition 3. Let the distance between a_i and a_{i+1} be $d_X(a_i, a_{i+1}) = \sqrt{\mathcal{G}_X(a_i) - \mathcal{G}_X(a_{i+1})| + z)}$, $i = 1, 2, ..., n-1$, where $z = 1$ if $a_{i+1} \in N_4(a)$ and $z = 2$ if $a_{i+1} \in N_8(a) \setminus N_4(a)$ The length of the path γ is defined by $\Lambda(\gamma) = \sum_{i=1}^{n-1} d_X(a_i, a_{i+1})$. The WDTOCS distance image is defined by

$$\mathcal{F}(x) = min(\Lambda(\gamma), \gamma \in \Psi_X(x,y)), \quad (3)$$

$$\mathcal{F}(y) = 0 \quad (4)$$

4 THE ALGORITHMS

The sequential algorithms require two images: the original gray-level image $\mathcal{G}(i,j)$ and a binary image $\mathcal{F}(i,j)$ which determines the region or regions in which the calculation is performed. $\mathcal{F}(i,j)$ does not have to be homogenious, it can consist of several disjoint regions with arbitrary shape and size. The symmetrical masks $m(k,l)$ are split into two or more masks. The principle of the Sequential Local Transform (SLT) is that the domain in scanned sequentially point by point in a prescribed raster pattern. The masks are passed over the image once each, split into forward and backward masks. (i,j) are coordinates in the images $\mathcal{G}(i,j)$ and $\mathcal{F}(i,j)$. (k,l) are coordinates in the mask $m(k,l)$. The length of one side of the rectangular image is denotes by size. The sequential algorithm for the DTOCS proceeds as follows:

Forward:
for i=(size+1)/2, ..., lines **do**
for j=(size+1)/2, ..., columns **do**

$$\mathcal{F}(i,j) = min(1 + \mathcal{F}(i+k, j+l) + d(k,l)) \qquad (5)$$

where $d(k,l) = |\mathcal{G}(i,j) - \mathcal{G}(i+k, j+l)|$,
$(k,l) \in$ *forward mask of mask m*

Backward:
for i=lines-(size-1)/2, ..., 1 **do**
for j=columns-(size-1)/2, ..., 1 **do**

$$\mathcal{F}(i,j) = min(1 + \mathcal{F}(i+k, j+l) + d(k,l)) \qquad (6)$$

where $d(k,l) = |\mathcal{G}(i,j) - \mathcal{G}(i+k, j+l)|$,
$(k,l) \in$ *backward mask of mask m*

The sequential algorithm for the WDTOCS proceeds as follows:

Forward:
for i=(size+1)/2, ..., lines **do**
for j=(size+1)/2, ..., columns **do**

$$\mathcal{F}(i,j) = min(1 + \mathcal{F}(i+k, j+l) + d(k,l)) \qquad (7)$$

where $d(k,l) = |\sqrt{\mathcal{G}(i,j) - \mathcal{G}(i+k, j+l) + m(k,l)}|$, $m(k,l)$ denotes the value of mask point (k,l) in the forward part of mask m.

Backward:
for i=lines-(size-1)/2, ..., 1 **do**
for j=columns-(size-1)/2, ..., 1 **do**

$$\mathcal{F}(i,j) = min(1 + \mathcal{F}(i+k, j+l) + d(k,l)) \qquad (8)$$

where $d(k,l) = |\sqrt{\mathcal{G}(i,j) - \mathcal{G}(i+k, j+l) + m(k,l)}|$, $m(k,l)$ denotes the value of mask point (k,l) in the forward part of mask m.
The values of mask points are as follows:
DTOCS: $a = 1, b = 1$
WDTOCS: $a = \sqrt{2.0}, b = 1.0$

The kernel masks $m(,k.l)$ used in this paper:

2-neighbor, 4-raster SLT

a	b	
	e	

Left to right, top to bottom.

	e	
	b	a

Right to left, bottom to up.

b	e	
a		

Left to right, bottom to top.

		a
	e	b

Right to left, top to bottom.

3-neighbor, 4-raster SLT

a	b	
b	e	

Left to right, top to bottom.

	e	b
	b	a

Right to left, bottom to top.

	b	a
	e	b

Right to left, top to bottom.

b	e	
a	b	

Left to right, bottom to up.

4-neighbor, 3-raster SLT

a	b	a
b	e	

Left to right, top to bottom.

b	e	
a	b	a

Left to right, bottom to top.

		a
	e	b
	b	a

Bottom to top, right to left.

4-neighbor, 4-raster SLT

a	b	a
b	e	

Left to right, top to bottom.

	e	b
a	b	a

Right to left, bottom to up.

a	b	a
	e	b

Right to left, top to bottom.

b	e	
a	b	a

Left to right, bottom to top.

4-neighbor, 2-raster SLT

a	b	a
b	e	

Left to right, top to bottom.

	e	b
a	b	a

Right to left, bottom to up.

5 RESULTS

It is typical for sequential local transform algorithms of distance transforms that, after the first round, the distance image contains erroneous distance values. Fortunately, as shown in [2], this problem can be overcome by applying more than one iteration rounds. This phenomenon holds also for gray-level image distance transforms as shown for the Distance Transform on Curved Space (DTOCS) and Weighted Distance Transform on Curved Space (WDTOCS) in [5].

The tests were performed using the "Leena" image of sizes 32×32, 64×64, 128×128, 256×256, and 512×512 pixels. Each pixel was presented by 8 bits. In Figure 1, a 2-neighbor, 4-raster algorithm for the DTOCS and the WDTOCS is tested. It shows how the number of erroneous pixels converges to zero after 3 iteration rounds for all image sizes. A similar result is also obtained for the WDTOCS. This is clearly faster than the convergence obtained using a conventional 4-neighbor, 2-raster algorithm, whose results are depicted in Figure 5 and Figure 6. The 3-neighbor, 4-raster algorithm gives assentially similar results for both the DTOCS and WDTOCS for all image sizes as the previous algorithm. See Figure 2. Figure 3 shows how the 4-neighbor, 3-raster algorithms converge. This neighbor-raster combination gives slightly better results than the previous ones. The number of erroneous pixels after the first iteration round is considerably smaller. Figure 4 shows the best convergence behavior which is obtained using the 4-neighbor, 4-raster algorithms for the DTOCS and the WDTOCS. In both the DTOCS and WDTOCS algorithms the number of erroneous pixels converges to zero already after the second iteration round. The behavior of the 2-neighbor, 4-raster and 3-neighbor, 4-raster algorithm is very near each other. See Figures 1 and 2. This resolution makes the curves look almost the same.

The 4-neighbor, 2-raster algorithm gives the worst performance because practically in all convex domains there are some minimal non-reversing paths which are not computed by either pass of the 2-raster algorithm. In [2] is is shown that 4 passes of the 4-raster algorithm is sufficient to compute distance along all non-reversing paths in binary images, because each pass correctly computes non-reversing paths lying in one quadrant. However, this does not exactly hold for gray-level images. In this paper it is shown, that also for gray-level images this neighbor-raster combination is the best, but 2 iteration rounds are needed to get an error-free distance image.

6 Conclusion

In this paper, new and more efficient algorithms for the Distance Transform on Curved Space (DTOCS) and the Weighted Distance Transform on Curved Space (WDTOCS) are presented. These algorithms produce distance images, in which the distamce value of each point is proportional to gray value differences in minimal paths. In [1] and [3] the distance values are proportional to gray values themselves.

In [2] it is shown that local sequential transform algorithms leave some pixels in the binary image with erroneous distance values after the first iteration round. Therefore they must be applied several times using the distance image as input. It is shown in [5] that this applies also to the DTOCS and WDTOCS algorithms which calculate distance transforms for a gray-level images. However, after a few iteration rounds the number of erroneous pixels converges to zero. It is shown in this paper that the new algorithms converge to an error-free distance image considerably faster than the older algorithms presented in [6], [5]. Furthermore, because of the raster scanning approach they are easily applied other image grids than the rectangular one, and are easy to implement.

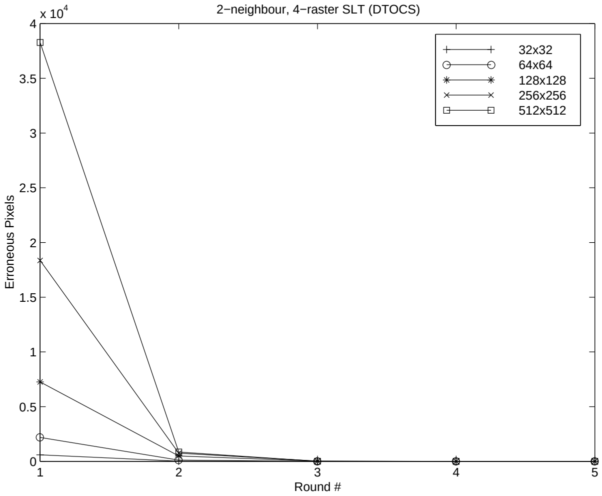

Figure 1. 2-neighbor, 4-raster algorithm results for the DTOCS and the WDTOCS.

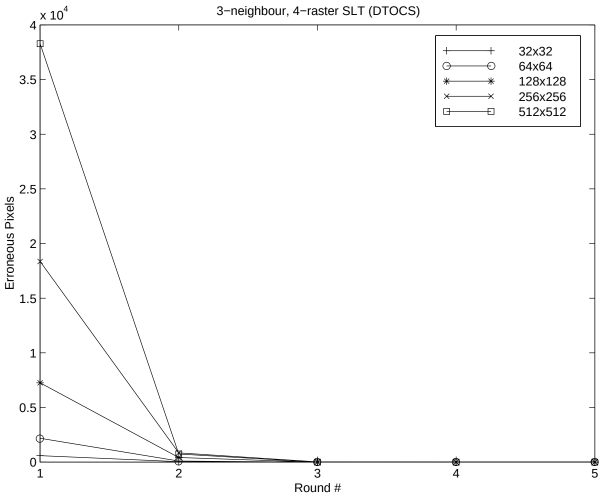

Figure 2. 3-neighbor, 4-raster algorithm results for the DTOCS and the WDTOCS.

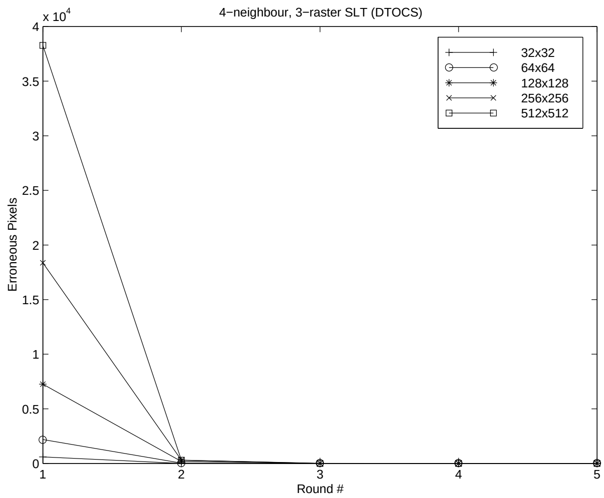

Figure 3. 4-neighbor, 3-raster algorithm results for the DTOCS and the WDTOCS.

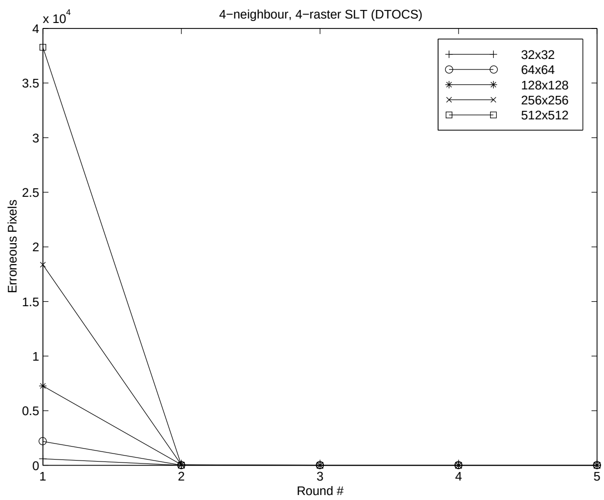

Figure 4. 4-neighbor, 4-raster algorithm results for the DTOCS and the WDTOCS.

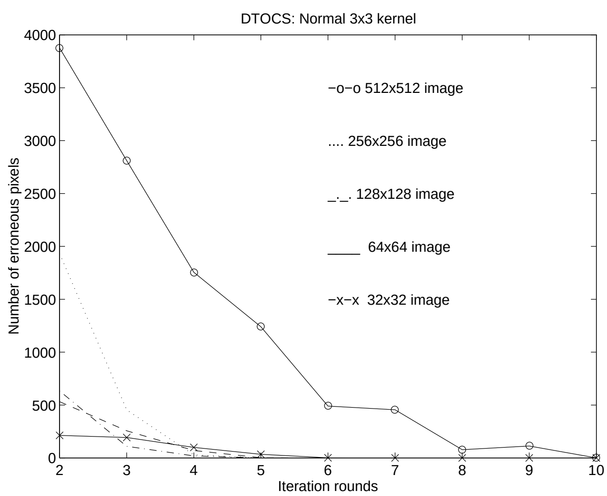

Figure 5. 4-neighbor, 2-raster algorithm results for the DTOCS.

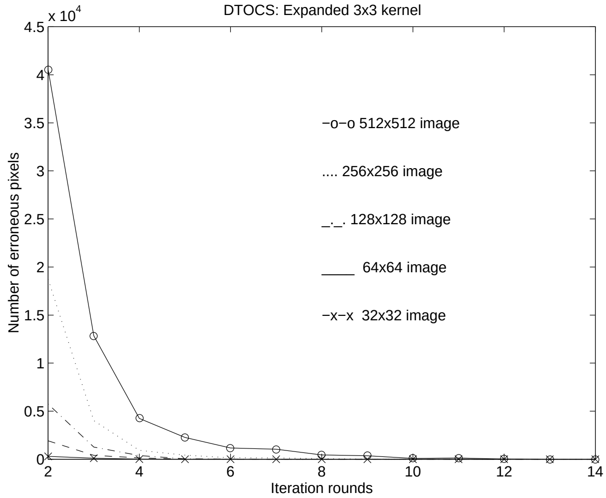

Figure 6. 4-neighbor, 2-raster algorithm results for the WDTOCS.

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