Synchronous Fiber-Optic CDMA Systems with Interference Estimators

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***Abstract*— Multiple-user interference estimation and cancellation techniques are proposed for synchronous optical code-division multiple-access (CDMA) communication systems. At the receiving end the multiple-user interference is estimated and is used to adapt a threshold level that is placed after an optical correlator. The special properties of the modified prime sequence codes are utilized in the estimation process. Four methods for adapting the threshold are proposed in this paper. The performance of the above system is analyzed and the bit error rates of the four adaptation schemes are compared to each other and to that of the system without cancellation. Our results reveal that the proposed system is very efficient in eliminating the effect of the multiple-user interference. Moreover, it is shown that the error floor which distinguishes the traditional systems is taken away under the proposed schemes.**

***Index Terms*—Code division multiple access (CDMA), direct-detection optical channel, interference cancellation, interference estimation, modified prime sequence codes, ON–OFF keying, optical CDMA.**

I. INTRODUCTION

RECENTLY interest has been bestowed to optical code-division multiple-access (CDMA) [1]–[15] for future very high speed optical networks. Optical CDMA has several advantages over optical time-division multiple-access (TDMA), e.g., complete utilization of the entire time-frequency domain by each subscriber, flexibility in network design (because the quality depends on the number of active users), and security against interception. Synchronous CDMA has an additional advantage over asynchronous CDMA, where the number of available code sequences (and in turn the number of subscribers) is much higher in the former under a given throughput constraint. The latter does not require, however, any time management as in the former. It follows that synchronous CDMA is suitable for very high speed networks with real time requirements (e.g., voice and digitized video). Crosswise, asynchronous CDMA is suitable for bursty traffic with no stringent time requirements (e.g., data transmission).

On the other hand, optical CDMA has a disadvantage over TDMA which is due to the multiple-user interference in the former. This leads in turn to a serious degradation in the bit error probability as the number of simultaneous users increases. This degradation cannot be overcome even for arbitrary high-optical power. In fact there will be an asymptotic error floor which limits the number of users that can communicate simultaneously and reliably.

Several interference cancellation techniques have appeared in literature aiming at lowering these asymptotic error floors. Salehi and Brackett [6] have used an optical hardlimiter that is placed before the optical correlator at the receiver side. This optical hardlimiter was shown to be able to remove some of the interference patterns. Ohtsuki *et al.* [9] have proposed a synchronous optical CDMA system with double optical hardlimiters placed before and after the optical correlator. It has been shown that this system introduces an improvement in the performance over the system with single optical hardlimiter as long as the number of users is not so large. In the case of asynchronous optical CDMA, Ohtsuki [11] has shown that this improvement continues for all possible number of users. In [12] Ohtsuki has been able to reduce the error floor even lower than that of the system with double hardlimiters. However the performance of the system in [12] is worse than that of the system with double hardlimiters if the optical power is not large enough. Lin and Wu [15] have suggested a synchronous optical CDMA system with an adaptive optical hardlimiter (or equivalently, a tunable optical attenuator) placed after the correlator receiver. They were able to show that the performance can be improved as compared to the system with double hardlimiters. In [10] and [13] we have proposed some cancellation techniques for both ON–OFF keying (OOK) and pulse-position modulation (PPM) CDMA systems. These techniques depend on estimating the interference from a knowledge of some other users' code sequences. In [14] we have developed a receiver model which performs refined observations at chip levels rather than at frame levels. We were able to show that using chip-level receivers the error floors can be lowered to that of the optimum receivers.

Unfortunately, in all the aforementioned cancellation techniques the effect of the multiple-user interference was not completely removed and error floors still exist when the number of users increases above a certain threshold. Thus for heavy load transmission, these cancellation techniques cannot provide reliable communication.

In this paper we propose a new interference cancellation technique in direct-detection synchronous optical OOK-CDMA. we make use of the special properties of the modified prime sequence codes [5] in order to estimate the interference to the desired user at the receiving end. The estimated interference is then used to adapt a threshold level that is

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placed after the optical correlator. We propose in this paper four simple methods for adapting this threshold. We then analyze the performance of the suggested system taking into account the effect of the Poisson shot noise processes of the photodetectors at the receiver. We neglect, however, the effect of both the dark current and the thermal noise since their effect is negligible in these types of systems. Our results reveal that the proposed system is very efficient in eliminating the effect of the multiple-user interference. Moreover, it is shown that the error floor which distinguishes the traditional CDMA systems can be easily taken away under the proposed schemes.

The remaining of this paper is organized as follows. The system description and the receiver model are given in Section II. The interference estimation method is introduced in Section III. Section IV is devoted for the derivation of the bit error probabilities and the description of the different types of cancellation techniques. In Section V we present some numerical results where we investigate the effect of some parameters (throughput, number of users, average power, etc.) on the performance of the proposed schemes. Comparisons with the system without cancellation are also presented in this section. Finally we give some concluding remarks in Section VI.

II. System Description

A. Optical CDMA Transmitter Model

Let a prime number p be given. A modified prime sequence code can be constructed according to [5]. There are p^2 code sequences that can be generated using this number. Each code sequence has a weight equals to p and a length p^2. This code can be divided into p groups, each group consists of p different code sequences. The cross-correlation function (C_{ij}) between code sequences i and j is given by

$$C_{ij} = \begin{cases} p; & \text{if } i = j, \\ 0; & \text{if } i \text{ and } j \text{ share the same group and } i \neq j, \\ 1; & \text{if } i \text{ and } j \text{ are from different groups.} \end{cases}$$

Each subscriber is assigned a code sequence called the address or signature. The last code in each group is never assigned to any user and is reserved for the multiple-user interference (MUI) estimation at the receiving end. This code sequence is assumed to be known to all users sharing its group. The total number of subscribers is thus equal to $p^2 - p$. Out of this number we assume that there are N active (simultaneous) users and the remaining $p^2 - p - N$ users are idle. Each active user transmits a signature sequence of p laser pulses (representing the destination address) over a time frame T_b if the data bit is "1." On the other hand, if the data bit is "0," no pulses are transmitted during the time frame. This time frame T_b is inversely proportional to the bit rate R_b. The laser pulsewidth is thus

$$T_c = \frac{T_b}{p^2} = \frac{1}{R_b p^2}.$$

All the optical signals from all active users are then combined together and transmitted across an optical network to the receiver. We define a random variable γ_j, $j \in \{1, 2, \ldots, p^2\}$, as follows

$$\gamma_j = \begin{cases} 1; & \text{if code } j \text{ is assigned to a user and} \\ & \text{this user is active,} \\ 0; & \text{otherwise.} \end{cases}$$

Thus

$$\sum_{j=1}^{p^2} \gamma_j = N.$$

Without loss of generality, we always assume that user one is the desired user ($\gamma_1 = 1$). Let the random variable T represent the number of active users in the first group:

$$T = \sum_{j=1}^{p} \gamma_j = 1 + \sum_{j=2}^{p-1} \gamma_j.$$

Assuming that the random variable γ_j is uniform, it is easy to check that the probability distribution of T, given that user 1 is active, can be written as

$$P_T(t) = \frac{\binom{p^2 - 2p + 1}{N - t}\binom{p-2}{t-1}}{\binom{p^2 - p - 1}{N - 1}}, \quad t \in \{t_{\min}, t_{\min} + 1, \ldots, t_{\max}\},$$

where

$$t_{\min} \stackrel{\text{def}}{=} \max\{1, N + 2p - p^2 - 1\}$$

and

$$t_{\max} \stackrel{\text{def}}{=} \min\{N, p - 1\}.$$

B. Optical CDMA Receiver Model

The block diagram of the desired user's receiver (with interference cancellation) is shown in Fig. 1. The received signal is split into two equal signals using a 1×2 optical splitter. The first signal is directed to the upper (main) branch, where it is correlated with the signature code sequence that characterizes the desired user. The correlator output is then photodetected, integrated, and sampled at the end of the time frame. The output of the sampler is proportional to the photon count (denoted by Y_1) collected over the bit duration T_b. The second signal is directed to the lower branch, where the interference estimation process is accomplished. In this branch the second signal is correlated with the last code sequence in the desired user's group, which was preserved *a priori*. the correlator output undergoes similar processing as in the main branch. The photon count collected over the time frame T_b from this branch will be denoted by Y_p. This photon count Y_p then passes through an interference estimator to provide an estimate (denoted by $\hat{\kappa}$) on the interference in the main branch. The decision on the transmitted data is accomplished in the main branch where Y_1 is compared to a threshold θ. The value of this threshold is dependent on both Y_p and $\hat{\kappa}$, and will be adjusted at the data rate before the decision is made.

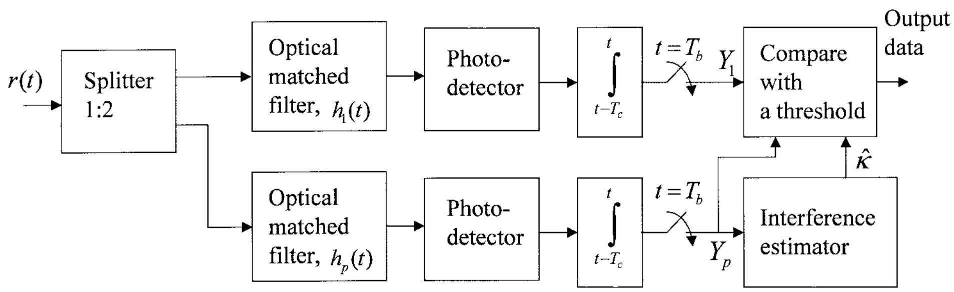

Fig. 1. Direct-detection optical OOK-CDMA system model with interference cancellation.

C. Statistics of the Interference Random Variable

Within the time frame T_b, the interference random variable κ can be written as

$$\kappa \stackrel{\text{def}}{=} \sum_{j=2}^{p^2} b_j C_{1j} \gamma_j$$

where $b_j \in \{0, 1\}$ denotes the binary data of the user which is assigned code sequence j. If code j is not assigned then b_j can be reset to 0. In our analysis we assume equiprobable binary data symbols. Since $C_{1j} = 0$ for any $j \in \{2, 3, \ldots, p\}$ and 1 for any $j \in \{p+1, p+2, \ldots, p^2\}$, then

$$\kappa = \sum_{j=p+1}^{p^2} b_j \gamma_j.$$

For equiprobable data symbols, the probability distribution of this random variable given $T = t$ is

$$P_{\kappa|T}(l|t) = \frac{1}{2^{N-t}} \binom{N-t}{l}, \qquad l \in \{0, 1, \ldots, N-t\}. \tag{1}$$

D. Statistics of the Decision and Estimation Random Variables

If PIN photodetectors are used then both the decision and estimation random variables (Y_1 and Y_p, respectively) can be modeled as Poisson random variables given that $\kappa = l$. Their conditional means (expected values) are

$$E[Y_1|\kappa = l] = Qpb_1 + Ql \quad \text{and} \quad E[Y_p|\kappa = l] = Ql \tag{2}$$

respectively. Here Q denotes the average photon count per pulse (calculated after the optical splitter). We assume without loss of generality that the optical matched filter does not add any attenuation to the received optical signals.

III. Interference Estimation

We provide here an interference estimator that depends on the maximum *a posteriori* probability (MAP) concept. Given $T = t$, the estimator output $\hat{\kappa}$ takes values in the finite set $\{0, 1, 2, \ldots, N-t\}$. Thus given $Y_p = i$, we decide that $\hat{\kappa} = m$ if

$$(\forall\, l \neq m) \quad \Pr\{\kappa = m | Y_p = i, T = t\} > \Pr\{\kappa = l | Y_p = i, T = t\},$$

where $l, m \in \{0, 1, 2, \cdots, N-t\}$. Using Baye's rule this is equivalent to

$$\Pr\{Y_p = i | T = t, \kappa = m\} P_{\kappa|T}(m|t) > \Pr\{Y_p = i | T = t, \kappa = l\} P_{\kappa|T}(l|t).$$

By substituting for the last probabilities we get

$$e^{-Qm} \frac{(Qm)^i}{i!} \cdot \frac{1}{2^{N-t}} \binom{N-t}{m} > e^{-Ql} \frac{(Ql)^i}{i!} \cdot \frac{1}{2^{N-t}} \binom{N-t}{l}.$$

Hence, we decide that $\hat{\kappa} = m$ if

$$(\forall\, l \neq m) \qquad Y_p \log\left(\frac{m}{l}\right) > Q(m-l) - \log\binom{N-t}{m} + \log\binom{N-t}{l}.$$

Dividing both sides by $\log(m/l)$, the last decision rule is equivalent to

$$\max_{l \in \{0, 1, \cdots, m-1\}} \Psi_{l,m} < Y_p \leq \min_{l \in \{m+1, m+2, \ldots, N-t\}} \Psi_{l,m}$$

where

$$\Psi_{l,m} \stackrel{\text{def}}{=} \frac{Q(m-l) - \log\binom{N-t}{m} + \log\binom{N-t}{l}}{\log\left(\frac{m}{l}\right)}.$$

We claim that the solution to the last maximization problem is given by

$$\max_{l \in \{0, 1, \cdots, m-1\}} \Psi_{l,m} = \frac{Q - \log \dfrac{N-t-m+1}{m}}{\log \dfrac{m}{m-1}}. \tag{3}$$

Proof: We can write $\Psi_{l,m}$ as shown in (3a) at the bottom of the next page. The right-hand side (RHS) of (3) is achieved by setting $l = m-1$ in (3a). Thus, it suffices to show that $\Psi_{l,m} \leq$ RHS of (3) for any $l \leq m-1$. But we can upper bound $\Psi_{l,m}$ as follows:

$$\Psi_{l,m} \leq \frac{Q(m-l) - (m-l) \log \dfrac{N-t-m+1}{m}}{\log \dfrac{m}{l}} = \frac{m-l}{\log \dfrac{m}{l}} \left[Q - \log \frac{N-t-m+1}{m}\right].$$

But $(m-l)/\log(m/l)$ is an increasing function in l. Hence its maximum is achieved for $l=m-1$. Substituting this in the last inequality shows that $\Psi_{l,m} \le$ RHS of (3). □

In a similar way, we can prove that the solution to that last minimization problem is

$$\min_{l\in\{m+1,m+2,\cdots,N-t\}} \Psi_{l,m} = \frac{Q-\log\dfrac{N-t-m}{m+1}}{\log\dfrac{m+1}{m}}. \quad \square$$

We conclude that the interference decision rule is: decide that $\hat{\kappa}=m$ if

$$\chi_m < Y_p \le \chi_{m+1},$$

where

$$\chi_m \stackrel{\text{def}}{=} \begin{cases} 0^-; & \text{if } m=0,\\ \infty; & \text{if } m=N-t+1,\\ \left[Q-\log\left(\dfrac{N-t-m+1}{m}\right)\right]\Big/\log\left(\dfrac{m}{m-1}\right); \\ \quad \text{if } m\in\{1,2,\cdots,N-t\}. \end{cases}$$

IV. Decision Rules and Bit Error Rates

A. The Data Decision Rule

A threshold θ, which depends on both Y_p and $\hat{\kappa}$, is set. If the received photon count (Y_1) is greater than this threshold, "1" is declared, otherwise "0" is declared to be transmitted. The probability of bit error is thus given by

$$P_e = \sum_{t=t_{\min}}^{t_{\max}} \sum_{l=0}^{N-t} P_{e|\kappa,T} P_{\kappa|T}(l|t) P_T(t) \tag{4}$$

where

$$\begin{aligned} P_{e|\kappa,T} &= \tfrac{1}{2}(P[E|0]+P[E|1]) \\ &= \tfrac{1}{2}(\Pr\{Y_1>\theta|b_1=0,\kappa=l,T=t\} \\ &\qquad +\Pr\{Y_1\le\theta|b_1=1,\kappa=l,T=t\}). \end{aligned} \tag{5}$$

Of course the optimal bit error rate can be obtained by finding the best threshold function $\theta=\Theta(Y_p,\hat{\kappa})$ that minimizes the last probability of error. In this case both $P[E|0]$ and $P[E|1]$ can further be evaluated as follows:

$$\begin{aligned} &P[E|0] \\ &= \Pr\{Y_1>\Theta(Y_p,\hat{\kappa})|b_1=0,\kappa=l,T=t\} \\ &= \sum_{m=0}^{N-t}\sum_{i=0}^{\infty} \Pr\{Y_p=i,\hat{\kappa}=m|b_1=0,\kappa=l,T=t\} \\ &\quad \times \Pr\{Y_1>\Theta(Y_p,\hat{\kappa})|b_1=0,Y_p=i,\hat{\kappa}=m,\kappa=l, \\ &\qquad T=t\} \\ &= \sum_{m=0}^{N-t}\sum_{i=0}^{\infty} \Pr\{Y_p=i|\kappa=l\}\Pr\{\hat{\kappa}=m|Y_p=i,\kappa=l\} \\ &\quad \times \Pr\{Y_1>\Theta(i,m)|b_1=0,\kappa=l\} \\ &= \sum_{m=0}^{N-t}\sum_{i=\lfloor\chi_m\rfloor+1}^{\lfloor\chi_{m+1}\rfloor} \Pr\{Y_p=i|\kappa=l\} \\ &\quad \times \Pr\{Y_1>\Theta(i,m)|b_1=0,\kappa=l\} \\ &= \sum_{m=0}^{N-t}\sum_{i=\lfloor\chi_m\rfloor+1}^{\lfloor\chi_{m+1}\rfloor} e^{-Ql}\frac{(Ql)^i}{i!} \sum_{j=\lfloor\Theta(i,m)\rfloor+1}^{\infty} e^{-Ql}\frac{(Ql)^j}{j!}. \end{aligned} \tag{6}$$

Similarly

$$\begin{aligned} P[E|1] &= \Pr\{Y_1\le\Theta(Y_p,\hat{\kappa})|b_1=1,\kappa=l,T=t\} \\ &= \sum_{m=0}^{N-t}\sum_{i=\lfloor\chi_m\rfloor+1}^{\lfloor\chi_{m+1}\rfloor} \Pr\{Y_p=i|\kappa=l\} \\ &\quad \times \Pr\{Y_1\le\Theta(i,m)|b_1=1,\kappa=l\} \\ &= \sum_{m=0}^{N-t}\sum_{i=\lfloor\chi_m\rfloor+1}^{\lfloor\chi_{m+1}\rfloor} e^{-Ql}\frac{(Ql)^i}{i!} \sum_{j=0}^{\lfloor\Theta(i,m)\rfloor} e^{-Q(p+l)} \\ &\qquad \times \frac{[Q(p+l)]^j}{j!}. \end{aligned} \tag{7}$$

We introduce here four different threshold functions which (although not optimal) improve the bit error rate of the system without cancellation significantly.

B. Canceler 1

We find a suitable threshold function as follows. Let the output of the interference estimator be $\hat{\kappa}$. Given that $Y_1=j$, we use the following decision rule: decide data bit $b_1=1$ was transmitted if

$$\Pr\{b_1=1|Y_1=j,\kappa=\hat{\kappa}\} > \Pr\{b_1=0|Y_1=j,\kappa=\hat{\kappa}\}$$

or equivalently

$$\Pr\{Y_1=j|b_1=1,\kappa=\hat{\kappa}\} > \Pr\{Y_1=j|b_1=0,\kappa=\hat{\kappa}\}.$$

By substituting for the last probabilities, we get

$$e^{-Q(p+\hat{\kappa})}\frac{[Q(p+\hat{\kappa})]^j}{j!} > e^{-Q\hat{\kappa}}\frac{(Q\hat{\kappa})^j}{j!}.$$

That is, we decide that data bit $b_1=1$ was transmitted if

$$Y_1 > \frac{Qp}{\log\left(1+\dfrac{p}{\hat{\kappa}}\right)}.$$

$$\Psi_{l,m} = \frac{Q(m-l)-\log\dfrac{(N-t-l)(N-t-l-1)\cdots(N-t-m+1)}{m(m-1)\cdots(l+1)}}{\log\dfrac{m}{l}}. \tag{3a}$$

Otherwise, a "0" is declared to be transmitted. Thus, for Canceler 1, the threshold is a function of $\hat{\kappa}$ only

$$\Theta_1(Y_p, \hat{\kappa}) = \frac{Qp}{\log\left(1 + \frac{p}{\hat{\kappa}}\right)}. \tag{8}$$

C. Canceler 2

The threshold in Canceler 1, being a function of $\hat{\kappa}$ only, motivates us to search for a linear function of $\hat{\kappa}$. From the properties of the log function, we notice that

$$Q\hat{\kappa} \le \frac{Qp}{\log\left(1 + \frac{p}{\hat{\kappa}}\right)} \le Qp + Q\hat{\kappa}.$$

The last two extremes are also consistent with the conditional mean values of Y_1 in (2) given that $\kappa = \hat{\kappa}$. This suggests the following linear threshold function of $\hat{\kappa}$:

$$\Theta_2(Y_p, \hat{\kappa}) = \frac{Qp}{2} + Q\hat{\kappa}. \tag{9}$$

D. Canceler 3

We wish to emphasize that $\hat{\kappa}$ is in fact a deterministic function of Y_p. That is a knowledge of Y_p will identify completely the value of $\hat{\kappa}$. Thus, the threshold is in fact a function of a single random variable Y_p. This motivates us to choose the simplest function of Y_p which is the linear function:

$$\Theta_3(Y_p, \hat{\kappa}) = \alpha + \beta Y_p.$$

The implementation of this canceler is in fact much more simpler than that of the two previous cancelers. Indeed the interference estimator (last block in the lower branch) can be removed from Fig. 1. Our aim in this subsection is to determine suitable constants α and β for this canceler. To get these constants we first derive an upper bound on the bit error rate for this canceler. As for the case of no cancellation we assume that the constants α and β are dependent on the difference $N - t$. Using Chernoff bound $P[E|0]$ can be upper bounded as follows. For any $z > 1$

$$\begin{aligned} P[E|0] &= \Pr\{Y_1 > \alpha + \beta Y_p | b_1 = 0, \kappa = l, T = t\} \\ &= \Pr\{z^{Y_1 - \beta Y_p} > z^{\alpha} | b_1 = 0, \kappa = l, T = t\} \\ &\le z^{-\alpha} \cdot E[z^{Y_1} | b_1 = 0, \kappa = l] \cdot E[z^{-\beta Y_p} | \kappa = l] \\ &= z^{-\alpha} \exp[Ql(z-1) - Ql(1 - z^{-\beta})]. \end{aligned}$$

Similarly

$$\begin{aligned} P[E|1] &= \Pr\{Y_1 \le \alpha + \beta Y_p | b_1 = 1, \kappa = l, T = t\} \\ &= \Pr\{z^{\beta Y_p - Y_1} \ge z^{-\alpha} | b_1 = 1, \kappa = l, T = t\} \\ &\le z^{\alpha} \cdot E[z^{-Y_1} | b_1 = 1, \kappa = l] \cdot E[z^{\beta Y_p} | \kappa = l] \\ &= z^{\alpha} \exp[-Q(p+l)(1 - z^{-1}) + Ql(z^{\beta} - 1)]. \end{aligned}$$

Choice of α and β: in view of

$$P_{e|\kappa, T} = \tfrac{1}{2}(P[E|0] + P[E|1])$$

we choose α so as to minimize the upper bound on the last sum. This can be achieved if

$$z^{2\alpha} = \exp[Qp(1 - z^{-1}) - Ql(z^{-1} - z^{-\beta}) + Ql(z - z^{\beta})].$$

Next we choose β so that α becomes independent of l. This can be achieved if $\beta = 1$. In this case α reduces to

$$\alpha = \frac{Qp}{2} \cdot \frac{1 - z^{-1}}{\log z}$$

and $P_{e|\kappa, T}$ can be upper bounded as

$$P_{e|\kappa, T} \le \exp\left[-\frac{Qp}{2}(1 - z^{-1}) + Ql(z + z^{-1} - 2)\right].$$

Let $z = 1 + \delta$, $\delta > 0$. Thus, $z^{-1} \le 1 - \delta + \delta^2$ and the last probability can further be upper bounded by

$$P_{e|\kappa, T} \le \exp\left[-\frac{Qp}{2}(\delta - \delta^2) + Ql\delta^2\right].$$

Whence

$$\begin{aligned} P_{e|T} &= \sum_{l=0}^{N-t} P_{e|\kappa, T} P_{\kappa|T}(l|t) \\ &\le \exp\left[-\frac{Qp}{2}(\delta - \delta^2)\right] \cdot \frac{1}{2^{N-t}}\left[1 + e^{Q\delta^2}\right]^{N-t} \\ &= \exp\left[-\frac{Qp}{2}(\delta - \delta^2) + Q(N-t)\delta^2\right] \\ &\quad \cdot \frac{1}{2^{N-t}}\left[1 + e^{-Q\delta^2}\right]^{N-t}. \end{aligned}$$

Now we select δ so as to minimize the exponent

$$-\frac{Qp}{2}(\delta - \delta^2) + Q(N-t)\delta^2. \tag{10}$$

This can be achieved for

$$\delta = \frac{p}{2p + 4(N-t)}. \tag{11}$$

Combining our results for the third threshold function, we obtain

$$\Theta_3(Y_p, \hat{\kappa}) = \alpha + Y_p \tag{12}$$

where

$$\alpha = \frac{Qp}{2} \cdot \frac{\delta}{(1+\delta)\log(1+\delta)}. \tag{13}$$

The exact bit error rate for this canceler can be obtained by substituting $\theta = \Theta_3(Y_p, \hat{\kappa})$ in (4) and (5).

E. Upper Bound on the Bit Error Rate of Canceler 3

We show here that the previous canceler does not exhibit any error floor. We start by completing the upper bound on the error probability in the last subsection. Substituting δ from (11) into (10), we get

$$-\frac{Qp}{2}(\delta - \delta^2) + Q(N-t)\delta^2 = -\frac{Qp}{4}\delta.$$

Hence

$$P_{e|T} \le \exp\left[-\frac{Qp}{4}\delta\right] \cdot \frac{1}{2^{N-t}}\left[1 + e^{-Q\delta^2}\right]^{N-t}$$

and

$$P_e = \sum_{t=t_{\min}}^{t_{\max}} P_{e|T} P_T(t). \tag{14}$$

But the right-hand sides diminish to zero as $Q \to \infty$, which proves that there is no error floor on the probability of error.

F. Canceler 4

The parameter α given in Canceler 3 is dependent on the difference $N-t$. This difference somehow should be conveyed to the receiver. Since the rate of change of this difference is negligible with respect to that of the data, no much information is carried in this difference. It can be transmitted as a burst signal to the receiver without appreciably affecting the transmission rate. Another way is to estimate its value from Y_p, but in this case the interference estimator should be installed again at the receiver and the simplicity of Canceler 3 is lost. We suggest here another canceler which retain the simplicity of canceler 3 and does not require a knowledge of $N-t$. We introduce the variable τ as follows:

$$\alpha = \frac{Qp}{2}\cdot\frac{\delta}{(1+\delta)\log(1+\delta)} = \frac{Qp}{2}\cdot\tau.$$

A careful study of τ shows that it does not change appreciably as $N-t$ changes. Indeed τ is a decreasing function in δ and is bounded above by "1." Since δ never exceeds 0.5 [cf., (11)], we have

$$0.822 \le \tau < 1.$$

This suggests a midway choice of $\tau = 0.9$. Hence, the fourth threshold function is chosen as follows:

$$\Theta_4(Y_p, \hat{\kappa}) = 0.45Qp + Y_p. \tag{15}$$

Comparing (8), (9), (12), and (15), it turns out that Canceler 4 provides the simplest hardware realization among the four cancelers because its threshold is independent of both $\hat{\kappa}$ and the difference $N-t$. Thus we can remove the interference estimator (last block in the lower branch) from Fig. 1 and we do not need to transmit any information about $N-t$. Canceler 3 provides the next simple hardware realization because the interference estimator can still be removed but the information on the difference $N-t$ should be transmitted to the receiver as described earlier. Canceler 1 is the most complex one because in addition to the dependence of its threshold on both $\hat{\kappa}$ and $N-t$, a logarithmic function is required for threshold evaluation at the receiver side.

V. Numerical Results

In this section we compare between the performance of the OOK-CDMA system with and without interference cancellation. Equations (4)–(15) are used in our bit error rates' calculations for the system with cancellation. The corresponding equations for the system without cancellation are provided in the Appendix. We evaluate our numerical results in Figs. 2 and 3 under a fixed throughput-pulsewidth product

$$R_0 = R_b T_c = 10^{-3} \quad \text{bits/chip.}$$

The prime number p is chosen as the maximum prime number that satisfies the above constraint on R_0, i.e., $p = 31$. This choice of R_0 provides a flexibility to increase the number of users N over a wide range ($1 \le N \le 930$). The bit error rates for these systems are plotted in Fig. 2 versus the average received photons per bit μ when the number of users $N = 50$.

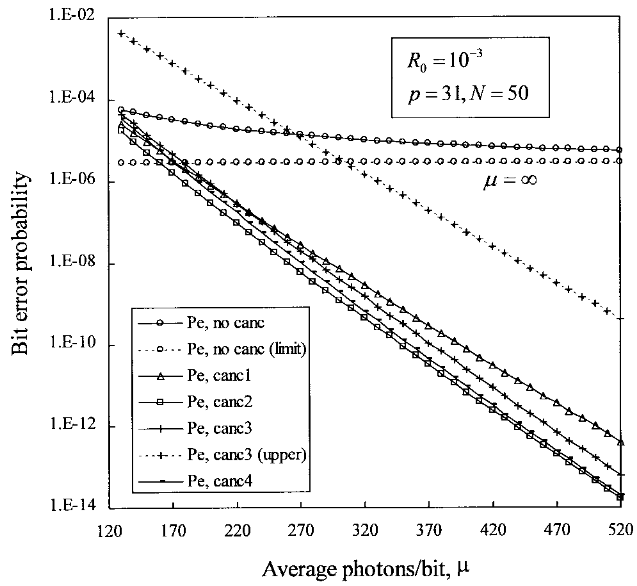

Fig. 2. A comparison between the bit error probabilities of the OOK-CDMA receivers (with and without interference cancellation) versus the average energy per bit.

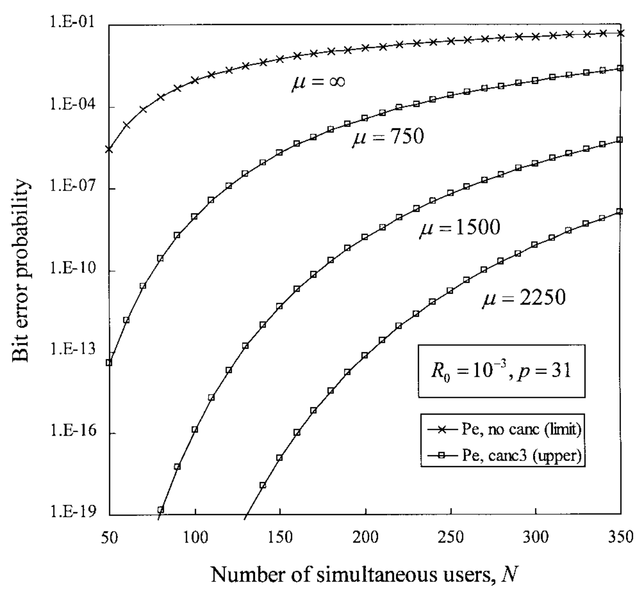

Fig. 3. A comparison between upper bounds on the bit error probabilities of the OOK-CDMA receiver with Cancelor 3 versus the number of simultaneous users for the different values of average energy per bit. The limiting error probability of the receiver with no cancellation is also shown.

Here $\mu = Qp$, where $2Q$ is the average received photons per pulse (before the splitter). The upper bound on the bit error rate of Canceler 3 and the limiting probability of error for the system without cancellation are also depicted in the same figure. It can be shown from this figure that the system without cancellation is not reliable (the error probability exceeds 10^{-9}) under the above throughput requirement even if we increased the average optical power without limit. With the implementation of any the proposed cancelers the performance improves significantly and reliable transmission is possible for very low energies (about 350 photons/b). Canceler 2 provides the best performance among the four cancelers, whereas Canceler 1

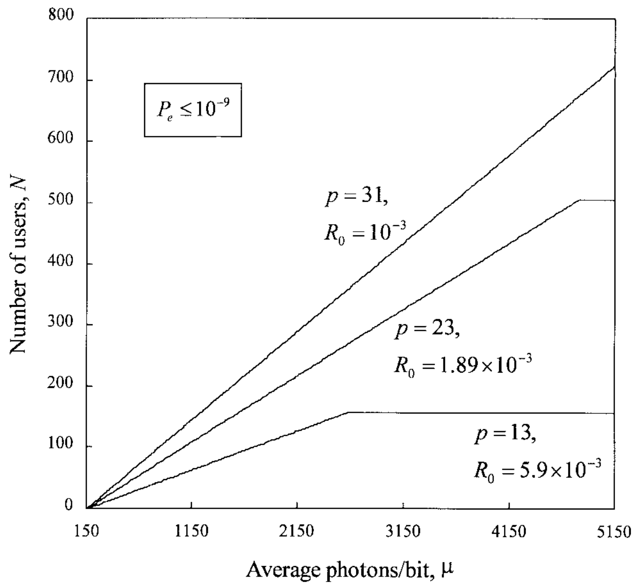

Fig. 4. A comparison between the maximum achievable number of users in OOK-CDMA receiver with Canceler 3 versus the average energy per bit for different values of throughput constraints and bit error rate not exceeding 10^{-9}.

is the worst among them. There is more than 12% save in energy when using Canceler 2 rather than Canceler 1 for an error rate of 10^{-9}. The gap between the error probabilities of Cancelers 1 and 2 gets wider as the energy increases. Canceler 3 is slightly worse than Canceler 2. However the gap between their error probabilities does not increase and the two curves are almost parallel to each other. The upper bound on the error rate of Canceler 3 is in fact an upper bound for all other cancelers as well. Interestingly, the bit error rate of Canceler 4 lies between that of Cancelers 2 and 3. Its performance is very close to Canceler 3 for low average power, whereas it becomes closer to Canceler 2 as the power increases.

In Fig. 3, we plot the upper bound on the bit error rates of Canceler 3 [cf., (14)] versus the number of users for different values of the average energy per bit when the throughput-pulsewidth product is held fixed. It is obvious that very large number of users can be accommodated in the optical CDMA channel with a reasonable and practical amount of optical energy. The corresponding probability of error for the system without cancellation is also plotted in the same graph with infinite optical energy. It is obvious that without cancellation the system is far from being useful when the number of users is large and/or the data rate is high.

In Fig. 4 we illustrate the maximum achievable number of users versus the average photons per bit. That is the number of users that can communicate reliably (with $P_e \leq 10^{-9}$) under a constraint on the throughput-pulsewidth product. In fact Fig. 4 provides lower bounds on the maximum achievable number of users since (14) has been used in our calculations. From this figure we can see that the maximum number of users is increasing linearly with the average energy per bit till it reaches a full capacity. This capacity is determined by the maximum number of available code sequences

$$N \leq p^2 - p$$

which in turn is limited by the constraint on the throughput-pulsewidth product

$$p \leq \frac{1}{\sqrt{R_0}}.$$

Of course, if this constraint is relaxed, more users can be loaded into the channel and still can communicate reliably by increasing the average power accordingly.

VI. Concluding Remarks

Four cancellation techniques have been proposed for synchronous direct-detection optical code-division multiple-access channels. The special grouping property of the modified prime sequence codes is the crucial promulgation in the design of these cancelers. The performance of the optical OOK-CDMA systems with and without the implementation of these cancelers have been analyzed and compared. From the theoretical analysis and the numerical results we can extract the following features of the proposed cancelers:

1) the implementations of these cancelers are rather simple. Canceler 4 provides the simplest hardware realization among the four;
2) Canceler 2 has the best performance in terms of the bit error rate. Its complexity is slightly better than Canceler 1, but worse than both Cancelers 3 and 4;
3) the multiple-user interference is not a limiting factor any more (the serious error floor has been purged) for the performance of the optical OOK-CDMA systems with the implementation of any of the proposed cancelers;
4) the limitation of these systems is only due to the requirements on the throughput and average power constraints.

Appendix
Optical CDMA Without Interference Cancellation

The system without interference cancellation is similar to the upper branch in Fig. 1. Since no splitter is used, the received optical energy per pulse is twice that of the system with cancellation, i.e., $2Q$. Further, all the available codes can be utilized and no code is reserved as in the cancellation case. Hence, the number of active users T in the desired group has a probability distribution as

$$P_T(t) = \frac{\binom{p^2-p}{N-t}\binom{p-1}{t-1}}{\binom{p^2-1}{N-1}}, \quad t \in \{t_{\min}, t_{\min}+1, \ldots, t_{\max}\},$$

where

$$t_{\min} \overset{\text{def}}{=} \max\{1, N+p-p^2\} \quad \text{and} \quad t_{\max} \overset{\text{def}}{=} \min\{N, p\}.$$

We assume (in favor of the system without cancellation) that the difference $N - t$ is known to the decision maker. The threshold is chosen so as to minimize the bit error rate given any difference $N - t$.

1) The Decision Rule: A threshold θ_{N-t} is set. If the received photon count (Y_1) is greater than this threshold, "1" is declared, otherwise "0" is declared to be sent. The probability of bit error is thus given by

$$P_b = \sum_{t=t_{\min}}^{t_{\max}} P_b^t P_T(t)$$

where

$$P_b^t = \tfrac{1}{2} \min_{\theta_{N-t}} (\Pr\{Y_1 > \theta_{N-t} | b_1 = 0, T = t\} + \Pr\{Y \leq \theta_{N-t} | b_1 = 1, T = t\}).$$

The last two probabilities can be evaluated as follows:

$$\begin{aligned}\Pr\{Y > \theta_{N-t} | b_1 = 0, T = t\} &= \sum_{l=0}^{N-t} P_{\kappa|T}(l|t) \Pr\{Y > \theta_{N-t} | b_1 = 0, \kappa = l, T = t\} \\ &= \sum_{l=0}^{N-t} \sum_{j=\lfloor \theta_{N-t} \rfloor + 1}^{\infty} \frac{1}{2^{N-t}} \binom{N-t}{l} e^{-2Ql} \frac{(2Ql)^j}{j!}.\end{aligned}$$

Similarly

$$\begin{aligned}\Pr\{Y \leq \theta_{N-t} | b_1 = 1, T = t\} &= \sum_{l=0}^{N-t} P_{\kappa|T}(l|t) \Pr\{Y \leq \theta_{N-t} | b_1 = 1, \kappa = l, T = t\} \\ &= \sum_{l=0}^{N-t} \sum_{j=0}^{\lfloor \theta_{N-t} \rfloor} \frac{1}{2^{N-t}} \binom{N-t}{l} e^{-2Q(p+l)} \frac{[2Q(p+l)]^j}{j!}.\end{aligned}$$

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