

# Analytical comparison of optical code-division multiple-access systems with and without a successive interference cancellation scheme using modified prime-sequence codes

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## 1 Introduction

Optical code-division multiple access (OCDMA) systems allow multiple users to transmit information over the same physical channel. However, the performance and capacity of CDMA systems are limited by multiple-access interference (MAI). In Ref. 1 we presented a technique inspired by radio multiplexing: the successive interference cancellation (SIC) scheme, a multiuser detection technique (MUD) that is a simple, attractive way to improve the system capacity. In Ref. 2 we analyzed the performance of SIC. It was found that the proposed SIC receiver effectively suppresses MAI and significantly improves the BER performance at each stage of the cancellation process. We have developed this technique further in Ref. 3, using a modified prime-sequence code, and shown that the prime-sequence code can suppress the MAI and increase the system capacity. In this paper we present a performance analysis of SIC and a comparison between optical CDMA systems with and without SIC using the modified prime sequence code.

**Abstract.** We analyzed direct-sequence code-division multiple-access system using a successive interference cancellation (SIC) scheme. Modified prime-sequence codes are utilized as signature sequences, and the performance measure studied in this paper takes into account the effect of imperfect interference cancellation. The basic principle of the SIC scheme is to subtract the strongest received signals from the original signal one by one till all users have been detected and demodulated. In this analysis we have compared optical code-division multiple-access systems with and without the cancellation scheme, and it is shown that the SIC scheme with  $-20$ -dBm effective power can suppress multiple-access interference better than the system without cancellation. © 2008 Society of Photo-Optical Instrumentation Engineers.

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Subject terms: Optical code-division multiple access (OCDMA); multiple-access interference (MAI); modified prime-sequence codes; successive interference cancellation (SIC).

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## 2 Direct-Sequence OCDMA System

### 2.1 Performance of the Modified Prime-Sequence Code

We consider an incoherent direct-sequence (DS) OCDMA system, which requires a  $(1, 0)$ -valued code sequence with small cross-correlation to mitigate multiple-user interference. On-off keying (OOK) modulation is used in this paper, and a prime-sequence code is employed. The construction of the code begins with a finite field (Galois field);<sup>4</sup> the rules for a finite field with a prime number ( $p$ ) of elements can be satisfied by carrying out the arithmetic modulo  $p$ . The prime-sequence code consists of the set of coded sequences of code length  $F=p^2$  derived from prime sequences of length  $p$ , where  $p$  is a prime number. Starting with the Galois field  $GF(p)=\{0, 1, \dots, j, \dots, p-1\}$ , each element  $s_{x,j}$  of a prime sequence  $S_x=(s_{x,0}, s_{x,1}, \dots, s_{x,j}, \dots, s_{x,p-1})$  is constructed by multiplying every element  $j$  from  $GF(p)$  by  $x$ , modulo  $p$ . The prime sequence is then mapped into a binary code sequence  $C_x=(c_{x,0}, c_{x,1}, \dots, c_{x,j}, \dots, c_{x,p^2-1})$  according to

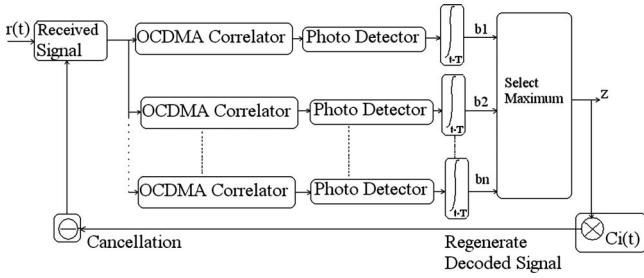


Fig. 1 SIC receiver block diagram.

$$C_{x,i} = \begin{cases} 1 & \text{for } i = s_{xj} + jp, \quad j = 0, 1, \dots, p-1, \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

When we take any two elements in the range 0 to  $p-1$ , and either add or multiply them, we take the result modulo  $p$ .<sup>4-6</sup> Let a prime number  $p$  be given. Then a modified prime sequence code can be constructed as follows:  $N=p^2$  code sequences can be generated, each of weight  $p$  and length  $p^2$ . According to Ref. 5, there are  $p$  distinct codewords for which the highest peak of periodic cross-correlation (i.e., the cross-correlation constraint  $\lambda_c$ ) is equal to one.<sup>7</sup> Under synchronized conditions, the cross-correlation function  $I_{xy}$  between any pair of code sequences  $x$  and  $y$  is given by<sup>7</sup>

$$I_{xy} = \begin{cases} p & \text{if } x = y, \\ 0 & \text{if } x \text{ and } y \text{ are in the same group,} \\ 1 & \text{if } x \text{ and } y \text{ are in different group.} \end{cases} \quad (2)$$

### 2.2 Successive Interference Cancellation Algorithm

The SIC works as follows: First, it detects and demodulates the strongest user signal currently present in the overall receiver signal. Then that signal is regenerated and subtracted from the total received signal, producing a new received signal. Then the algorithm repeats with the strongest user signal in the new received signal. A block diagram showing SIC is given in Fig. 1, and the general algorithm for it is as follows:<sup>3</sup>

- I. Recognize the strongest signal (the one with maximum correlation value).
- II. Decode the strongest signal.
- III. Regenerate the strongest signal, using its chip sequence.
- IV. Cancel the strongest signal by subtraction.
- V. Repeat until all users are decoded or a permissible number of cancellations are achieved.

### 2.3 DS OCDMA Performance Analysis

We consider here a simple way to suppress MAI, thereby improving the system's capacity and significantly improving its BER performance. Let each user be assigned a unique prime sequence code of length  $p^2$  and weight  $p$ , and let the total number of users be  $N=p^2$ . The received signal  $r(t)$  at the front end of the receiver is modeled as follows:

$$r(t) = \sum_{n=1}^{N=p^2} A_n b_n(t - \tau_n) \sum_{i=0}^{p^2-1} c_n^i(t - \tau_n) + n(t), \quad (3)$$

where

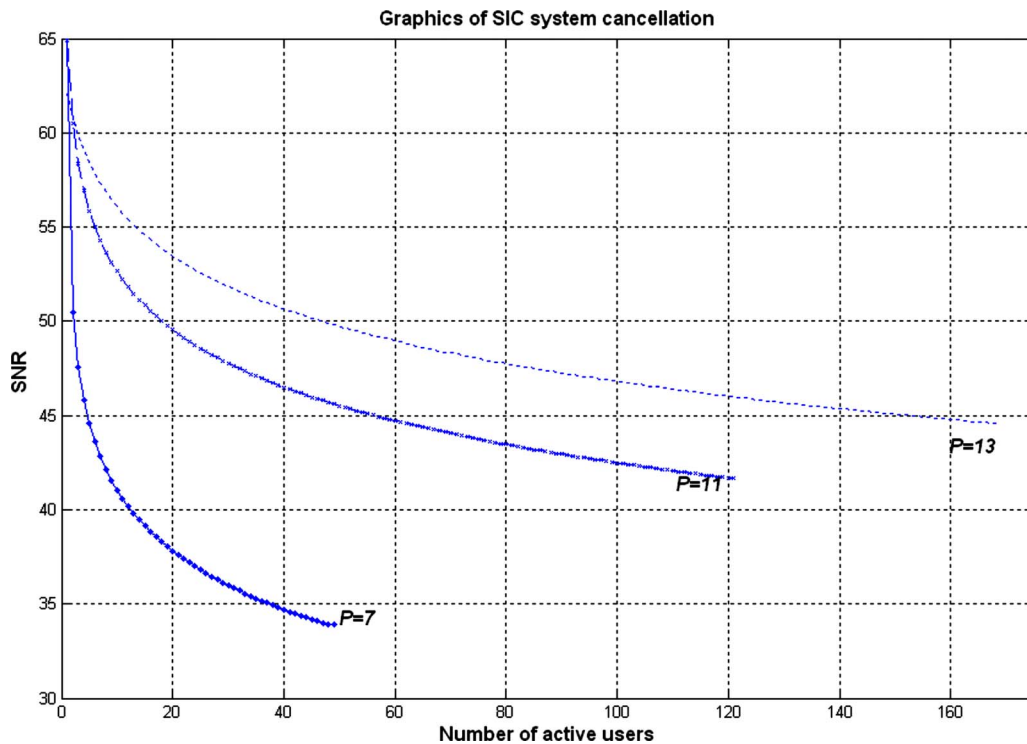


Fig. 2 SNR versus number of active users.

**Table 1** Typical parameters for the system.

Parameter	Value
Operating frequency	193.1 THz
Photodetector quantum efficiency	0.6
Receiver noise temperature	300 K
Receiver load resistance	1030 $\Omega$
Noise-equivalent electrical bandwidth	80 MHz

- $N$  = total number of users  
 $A_n$  = signal strength of the  $n$ th user  
 $b_n(t)$  = bit sequence of the  $n$ th user  
 $c_n(t)$  = spreading chip sequence of the  $n$ th user  
 $n(t)$  = noise signal (thermal noise)  
 $p^2$  = code length  
 $\tau_n$  = time delay of the  $n$ th user.

The receiver signal is fed into a bank of receivers, one for each user. The sign of the output of each receiver is the corresponding user decision. Detailed analysis of the SIC scheme for an incoherent DS OCDMA system can be found in Refs. 1 and 3. After the  $j$ 'th cancellation, the decision variable for the  $(j+1)$ th user, taking into account the code properties indicated in Eq. (1), is given by

$$Z_{j+1} = \zeta_{j+1} + l_{j+1}, \quad (4)$$

where

$$\zeta_{j+1} = \frac{A_{j+1}b_{j+1}}{p}, \quad (5)$$

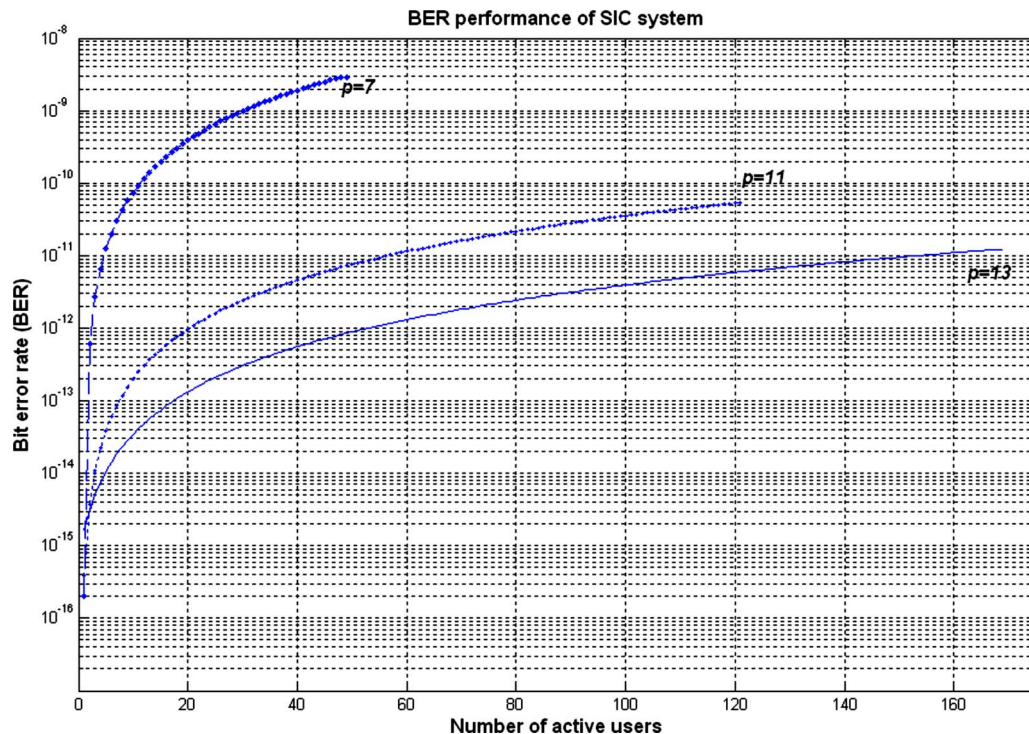
$$l_{j+1} = \frac{1}{p^2} \left[ \sum_{n=j+2}^{N=p^2} A_n b_n I_{n,j+1}(\tau_{n,j+1}) - \sum_{i=1}^j l_i I_{i,i+1}(\tau_{i,i+1}) \right] + n(t), \quad (6)$$

where the first term is a combination of the MAI of the uncanceled users and cumulative noise from imperfect cancellation.

The original prime code is analyzed by using a Gaussian approximation. The negative effects of shot noise, the receiver's dark current, and other sources of noise are ignored in order to focus only on the interference (the MAI) created by other simultaneous users. For the thermal noise, we can express the variance of  $l_{j+1}$  conditioned on  $A_n$  as follows:<sup>3</sup>

$$\Gamma_{j+1}^2 |_{\text{MAI}} = \frac{1}{p^2} \left\{ \sum_{n=j+2}^{N=p^2} A_n^2 b_n^2 \text{var}[I_{n,j+1}(\tau_{n,j+1})] + \sum_{i=1}^j \Gamma_i^2 \text{var}[I_{i,i+1}(\tau_{i,i+1})] \right\}. \quad (7)$$

We considering an ideal synchronous case, i.e.,  $\tau_n=0$  for all  $n$ . It has been shown<sup>8</sup> that the synchronous case is the worst case. Then the variance of the cross-correlation in Eq. (2) is given by:<sup>3</sup>


**Fig. 3** BER versus number of active users for ideal power (-20 dBm).

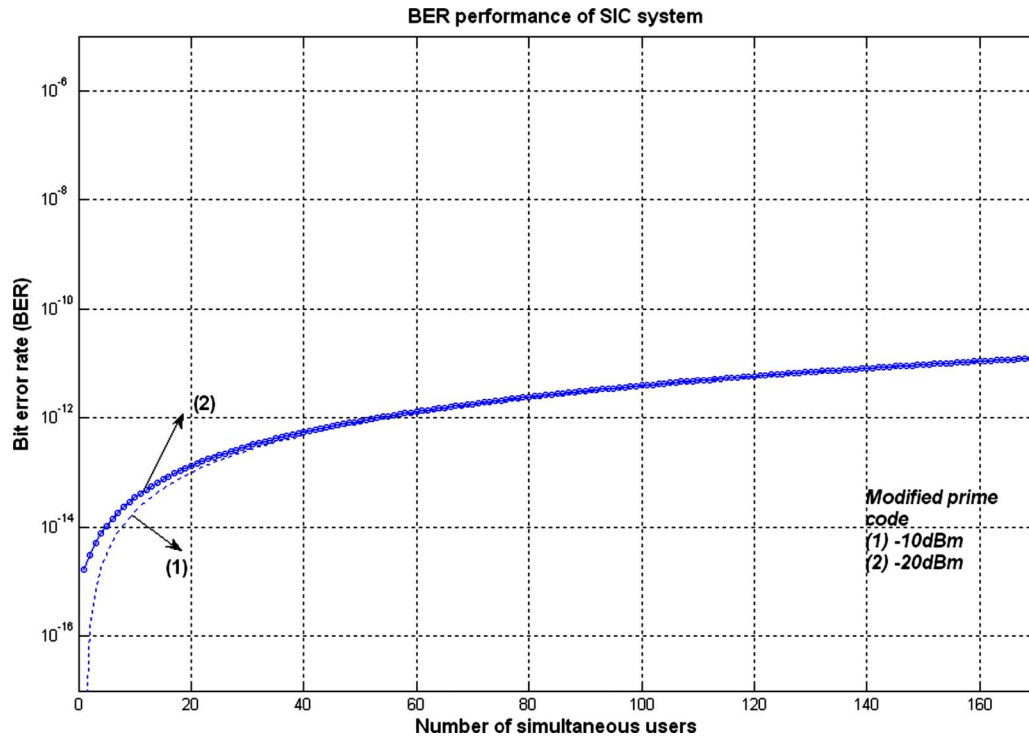


Fig. 4 BER versus number of active users for different powers.

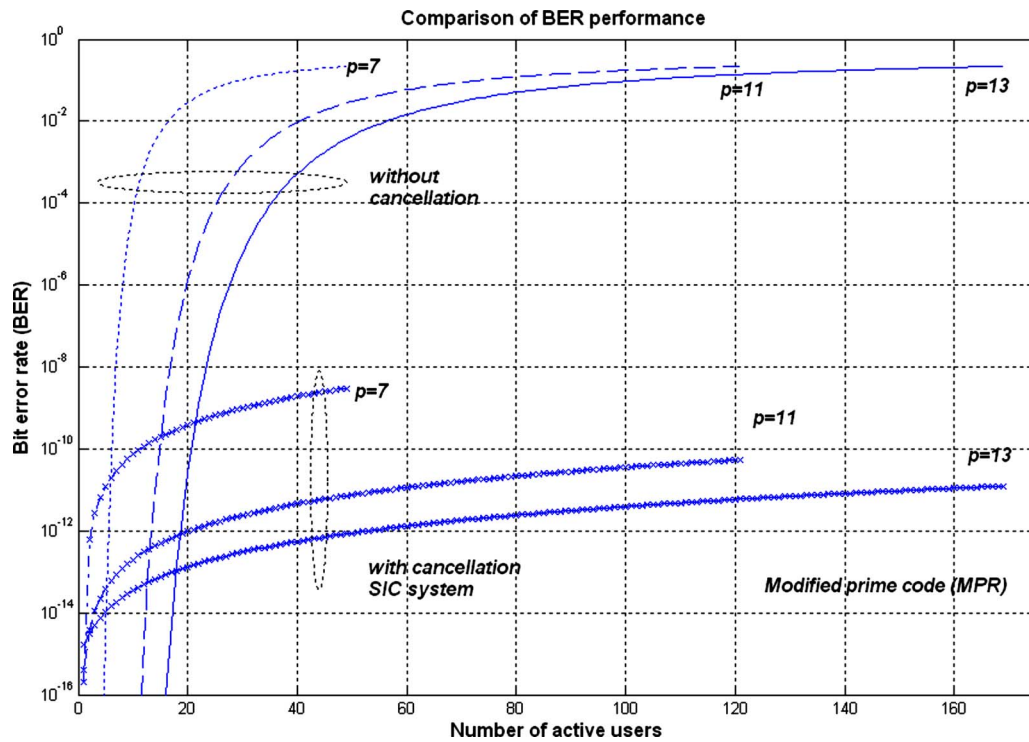


Fig. 5 Comparison of BER performance.



$$\text{var}(I_{n,j+1}) \approx \frac{1}{p^2}. \quad (8)$$

Substituting Eq. (8) and (7), we get the variance of the noise in the decision variable:

$$\Gamma_{j+1}^2|_{\text{MAI}} = \frac{1}{p^4} \left( \sum_{n=j+2}^{N=p^2} A_n^2 + \sum_{i=1}^j \Gamma_i^2 \right). \quad (9)$$

For the thermal noise a Gaussian distribution effect is considered:

$$\sigma_{\text{th}} = 4K_b T_n \frac{B}{R_L}, \quad (10)$$

where  $B$  is the noise-equivalent electrical bandwidth of the receiver in hertz;  $K_b$  is Boltzmann's constant,  $1.38 \times 10^{-23} \text{ J K}^{-1}$ ;  $T_n$  is the absolute receiver noise temperature in kelvins; and  $R_L$  is the receiver load resistor in ohms.

The responsivity of the photodetectors is given by  $\mathfrak{R} = \eta e / h \nu_c$ . Here,  $\eta$  is the quantum efficiency,  $e$  is the electron's charge,  $h$  is Planck's constant, and  $\nu_c$  is the central frequency of the original broadband optical pulse. Then from Eqs. (4), (9), and (10), we can get the signal-to-noise ratio function of  $A_n$  as follows:

$$\text{SNR}_{j+1} = \frac{\frac{\mathfrak{R}^2 A_{j+1}^2}{p^2}}{\frac{\mathfrak{R}^2}{p^4} \left( \sum_{n=j+2}^N A_n^2 + \sum_{i=1}^j \Gamma_i^2 \right) + 4K_b T_n \frac{B}{R_L}}. \quad (11)$$

To calculate the BER, we assume the noise  $I_{j+1}$  is Gaussian with mean zero and variance  $\Gamma_{j+1}^2$ . Then the probability of error, conditioned on the  $A_n$ , is as follows:

$$\text{BER}_{j+1} = Q \cdot \frac{|\zeta_{j+1}|}{\Gamma_{j+1}} \quad (12)$$

$$= Q \cdot \frac{\mathfrak{R} A_{j+1} b_{j+1}}{p} \left[ \frac{\mathfrak{R}^2}{p^4} \left( \sum_{n=j+2}^N A_n^2 + \sum_{i=1}^j \Gamma_i^2 \right) + 4K_b T_n \frac{B}{R_L} \right]^{1/2} \quad (13)$$

$$= Q \cdot \text{SNR}_{j+1}^{1/2}. \quad (14)$$

### 3 System Performance Results

In this section, we present analytical results of our mathematical model. Figure 2 shows the relation between the number of active users and the signal-to-noise ratio, using modified prime (MPR) codes for  $p=7$ ,  $p=11$ , and  $p=13$ . Table 1 gives the typical parameters used in our calculations. It can be seen that, using the effective power for each user (assumed ideal at  $-20$  dBm), each curve ends at the point where the number of active users is equal to the code size. The SNR increases significantly with increasing  $p$ .

**Remark.** For the same number of active users (40), the

SNR performance is increased by increasing  $p$ .

Figure 3 shows variations of the BER with the number of active users for  $p=7$ ,  $p=11$ , and  $p=13$ , assuming ideal power for all users ( $-20$  dBm). It is obvious that a large prime  $p$  improves the BER.

Figure 4 shows the relation between the number of active users and the SNR for our proposed SIC system, using MPR codes at  $p=13$ . It is seen, as expected, that the performance of the system is better with higher input effective power. We also see that the BER is more affected by noise at lower input power, as expected.

Figure 5 shows the BER performance comparison of the system with and without cancellation, for various numbers of active users. The results show that the cancellation scheme has better performance than the conventional scheme. Assuming that the prime number set as the length is 13, we can notice that for similar BER performance ( $10^{-12}$ ), less than 20 users can be active with conventional scheme, but that can be increased to 60 users with SIC cancellation scheme, giving a substantial increase in capacity.

### 4 Conclusion

In this paper, it has been shown that the SIC is a low-complexity suboptimal multiuser detector for CDMA systems. The major problem in an SIC system is the accumulated cancellation noise. An analysis of the performance of this system was presented under a modified prime-sequence code. The BER performance was significantly improved, and it improves further as the prime  $p$  increases. The system shows much lower BER with SIC cancellation than without cancellation. Hence, many more users can be accommodated by our proposed system.

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