Optical CDMA with Overlapping PPM

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Abstract—**Direct-detection optical code-division multiple access (CDMA) systems employing overlapping pulse-position modulation (OPPM) schemes are proposed. A union upper bound on the bit error rate is derived taking into account the effect of both multiple-user interference and receiver shot noise. The photodiodes' dark currents are neglected in order to have some insights on the problem. Performance characteristics are then compared to the traditional on off keying (OOK) and pulse-position modulation (PPM) schemes. It is shown that under the constraints of both throughput and chip time, OPPM system superperforms both traditional systems.**

Index Terms—**Optical CDMA, code division multiple access, on-off keying, pulse-position modulation, overlapping pulse-position modulation, direct detection optical channel, spread spectrum.**

I. Introduction

In the recent few years, an increasing interest has been given to the design and analysis of optical communication networks. This results from the enormous development of data communication networks all over the world and the impressive amount of data to be transmitted over these networks. Indeed in order to support image-based and multimedia applications, local and wide area networks must be able to endow data at rates of hundreds of megabits/second to desktops. Current monotonous networks will not accommodate such a large amount of data in the future. However, contemporary technology in optical fiber networks makes it possible to endow immense amount of transmission capacity economically.

In optical data communication networks, multiple access may be implemented by many methods such as time-division multiple-access (TDMA), wavelength-division multiple-access (WDMA), code-division multiple-access (CDMA), etc. CDMA has several advantages over other techniques. Namely, unnecessary time synchronization and frequency management, simple communication protocols, complete utilization of the entire time-frequency domain by each subscriber, flexibility in network design, and security against interception. One limitation, however, of binary CDMA is that it has less capacity than TDMA.

Optical CDMA systems with either binary on-off keying (OOK) or M-ary pulse-position modulation (PPM) schemes have been appeared in literature [15]–[8]. In [3], Dale and Gagliardi suggested encoding the data symbols using PPM format and then transmitting an aperiodic signature in place of the PPM optical pulse. They showed that under fixed throughput and chip time, there is no advantage in using PPM in place of OOK. On the other hand, they showed that PPM is superior to OOK if the average power rather than the chip time is the constraining factor.

In [5] and [6] we have remarked two main advantages of PPM-CDMA over OOK-CDMA:

i) Under bit error rate constraint, the maximum number of simultaneous users can not be increased, in the case of OOK-CDMA, without increasing the average power. In the case of PPM-CDMA, however, we can increase this number by increasing M and preserving the average power fixed.

ii) Even if we increased the average power we might still not be able to accommodate (*simultaneously*) all possible subscribers in the case of OOK. However for PPM we can accommodate any number of subscribers by increasing M.

Of course these advantages are obtained at the expense of decreasing the chip time which in turn increases the complexity of the system.

Recently, interest has been given to overlapping-pulse-position modulation (OPPM) as an alternative signaling format to the conventional pulse-position modulation in direct-detection optical channels [9]–[14]. This type of signaling can be considered as a generalization to PPM signaling, where overlapping is allowed between pulse positions. The reason to prefer OPPM over PPM is that the throughput (nats/s) can be improved without decreasing the pulsewidth. Moreover, OPPM retains the advantages of PPM in terms of implementation simplicity. Indeed the transmitter involves only time delaying of the optical pulse, and the receiver does not require knowledge of the signal or noise power.

In this paper we suggest employing OPPM in optical CDMA channels. Our aim is to tolerate both the throughput limitation of PPM-CDMA and the capacity limitation of OOK-CDMA systems.

We also compare between the bit error rate performance of the suggested scheme with both the traditional on off keying (OOK) and pulse-position modulation (PPM) schemes.

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In our theoretical analysis we consider the effect of both multiple-user interference and receiver shot noise. In order to have some insight on the results obtained we assume chip-synchronous uniformly-distributed relative delays among the transmitters and perfect photon counting processes at the receivers.

In the numerical analysis, we employ optical orthogonal codes (OOC's) [15], [16] as the signature code sequences. To have minimal interference between the users we adopt OOC's with periodic cross-correlations and out-of-phase periodic auto-correlations that are bounded only by 1.

The remaining of our paper is organized as follows. Section II is devoted for the description of our suggested system. Section III is devoted for the derivation of the bit error probability for the aforementioned system. In Section IV we present some numerical results where we investigate the effect of some parameters on the performance of the optical OPPM-CDMA system. Comparisons with other systems are also presented. Finally, we give our conclusion in Section V.

II. Optical OPPM-CDMA System Description

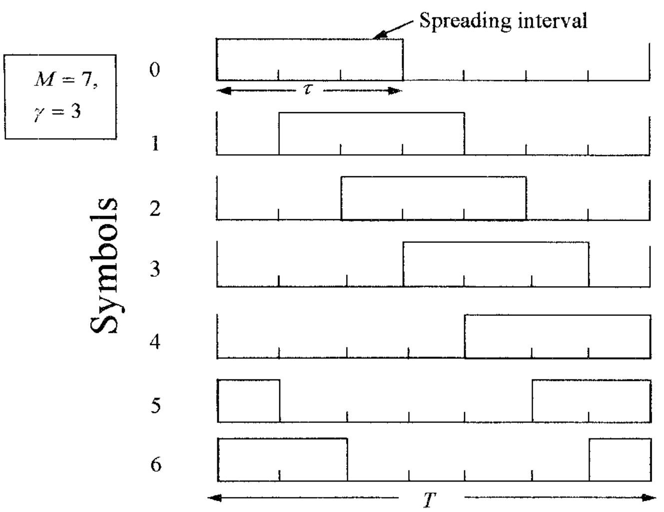

Fig. 1. An example of OPPM signal formats and their spreading intervals of a single user with M=7 and γ=3. Laser pulses are signaled at the leading edge of each spreading interval.

A. OPPM Signal Formats

In optical OPPM channel, with multiplicity M and index of overlap $\gamma \in \{1, 2, \ldots, M\}$, the information is conveyed by the position of a laser pulse of duration T_c, called the chip time, within a time frame of width T. Each laser pulse has an allowable spreading interval of duration τ, Fig. 1. This spreading interval will sometimes be called a slot. Each slot is subdivided into γ smaller subintervals of width τ/γ each. An overlap with depth $(1 - \frac{1}{\gamma})\tau$ is allowed between any two adjacent spreading intervals. There are M possible positions within the time frame. A transmitted pulse is said to be in position x, $x \in \{0, 1, 2, \ldots, M-1\}$, if it is initiated at time $x\frac{\tau}{\gamma}$. The corresponding spreading interval starts at the same instant of the laser pulse and ends τ s later, Fig. 1. To simplify the analysis we allow cyclic shifts to occur, that is if for some x it happened that $x\frac{\tau}{\gamma} + \tau > T$, the spreading interval is wrapped to the beginning of the frame (cf. Symbols 5 and 6, Fig. 1.) The number of disjoint slots within a time frame is equal to M/γ. The relation between T, γ, M, and τ is thus

$$T = \frac{M}{\gamma}\tau, \qquad \gamma \in \{1, 2, \ldots, M\} .$$

It is remarkable that PPM is a special case of OPPM when $\gamma = 1$.

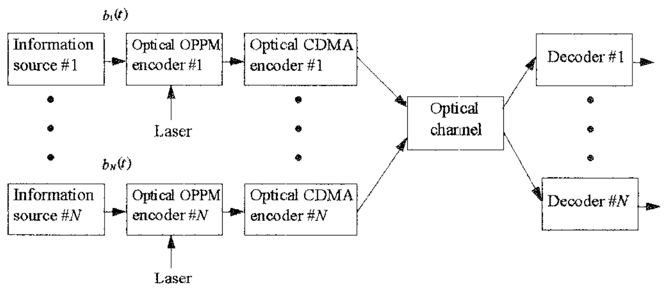

Fig. 2. An optical OPPM-CDMA system model.

B. Optical OPPM-CDMA System Model

The model for an optical OPPM-CDMA communication system is shown in Fig. 2 The transmitter is composed of N simultaneous information sources or users. Each user transmits M-ary continuous data symbols. The output symbol of the kth information source modulates the position of a tall narrow laser pulse, of width T_c, to form the OPPM initial signal. This signal is then passed to the CDMA encoder where it is then spread into w shorter laser pulses with same width T_c. Each encoder has its own signature sequence (of length L and weight w) which characterizes the kth user. The output waveform is finally transmitted over the optical channel.

A traditional way to achieve optical spreading is to use an optical tapped delay line which is composed of a splitter, delayers, and a combiner, Fig. 3. Wrapped OPPM-CDMA signals need special techniques for generation, Fig. 4. An example of the transmitted signal formats of a single user is shown in Fig. 5.

At the receiving end, the received optical signal (composed of the sum of N delayed users' optical signals) is correlated with the same signature sequence which characterizes

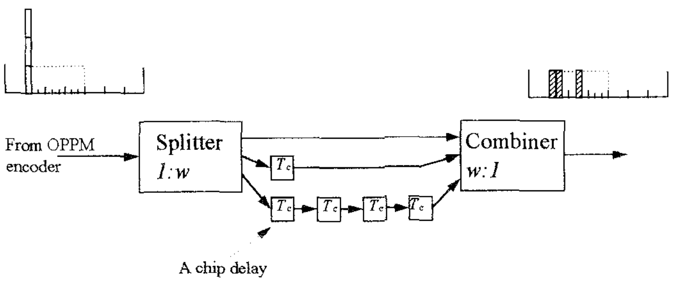

Fig. 3. An example of an optical CDMA encoder for one source. A signature code of 110010000 is assumed.

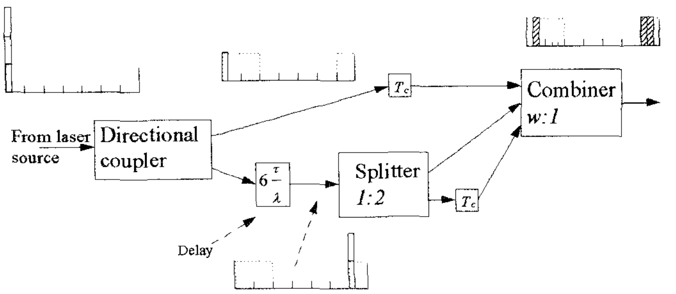

Fig. 4. An example of optical OPPM and CDMA encoders for a wrapped signal (Symbol 6 of Fig. 1.) A signature code of 110010000 is assumed.

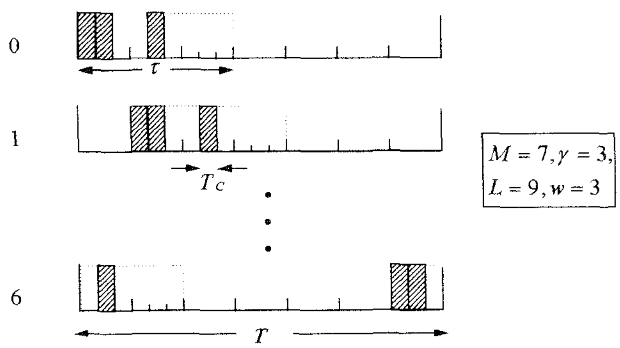

Fig. 5. An example of the transmitted signal formats of a single user in an OPPM-CDMA system with M=7, γ=3, L=9 and w=3. A signature code of 110010000 is assumed.

the desired user and then converted (using a photodetector) into an electrical signal which is passed to the OPPM decoder to obtain the data. The correlator can also be an optical tapped delay line, whereas the OPPM decoder is merely a comparison between the photon counts collected over the M time slots: the number of the slot with the greatest count is declared to be the transmitted symbol.

III. Bit Error Rate of the Optical OPPM-CDMA Systems

A. The Decision Rule

Each receiver counts the photons collected in the permissible chips (determined by its signature code) of every slot within the time frame. The number of the slot having the largest count is declared to be the transmitted symbol. We denote the photon count collected in slot $i \in \{0, 1, \ldots, M-1\}$ by Y_i. Symbol i is thus declared to be the true one if $Y_i > Y_j$ for every $j \neq i$. Hence, for equiprobable data symbols, the probability of error can be written as

$$P[E] = \sum_{i=0}^{M-1} P[E|i] \Pr\{D = i\} = \frac{1}{M} \sum_{i=0}^{M-1} P[E|i] \ ,$$

where D is a random variable that denotes the transmitted data symbol and

$$P[E|i] = \Pr\{Y_j \geq Y_i, \ \text{some } j \neq i | D = i\} \ .$$

It is obvious, because of the symmetry of the channel, that the last probability is independent of i. Whence

$$P[E] = \Pr\{Y_j \geq Y_0, \ \text{some } j \neq 0 | D = 0\} \ . \tag{1}$$

B. The Probability of Interference

Denote by κ_i, $i \in \{0, 1, \ldots, M-1\}$, the number of other users that cause interference in slot i with the desired user. Moreover denote the vector $(\kappa_0, \kappa_1, \ldots, \kappa_{M-1})$ by κ_0^{M-1}. It is easy to check that κ_0^{M-1} admits a multinomial distribution with parameters $N-1, p$:

$$\Pr\{\kappa_0^{M-1} = l_0^{M-1}\} = \frac{(N-1)!}{l_0! l_1! \cdots l_{M-1}! S!} \cdot p^{N-1-S} (1 - Mp)^S \ ,$$

where p denotes the probability that a single user causes an interference with the desired user at one pulse position, l_0^{M-1} is a realization vector for κ_0^{M-1}, i.e., $l_0^{M-1} = (l_0, l_1, \ldots, l_{M-1})$, and

$$S \stackrel{\text{def}}{=} N - 1 - \sum_{i=0}^{M-1} l_i \geq 0 \ .$$

The following proposition gives a characterization of the probability p.

Proposition 1: In a frame-level-synchronous optical OPPM-CDMA channel employing OOC's with weight $w > 1$, length $L \geq w^2$, and auto- and cross-correlation constraint $\lambda < 1$, if p denotes the probability that a single user interferes with the desired user at one pulse position then, for $L/\gamma =$ an integer,

$$p = \gamma \frac{w^2}{ML} ,$$

where M and γ denote the pulse position multiplicity and index of overlap, respectively.

Proof: If the data transmitted by the interfering user is within a similar slot as that transmitted by the desired user, an interference with probability $\frac{w^2}{ML}$ occurs on the average. If the active slot of the interfering user overlaps by $(1 - \frac{i}{\gamma})\tau$ with that of the desired user, an interference with probability $\frac{w^2}{ML}(1 - \frac{i}{\gamma})$ occurs on the average. Thus

$$p = \sum_{i=-\gamma}^{\gamma} \frac{w^2}{ML}\left(1 - \frac{|i|}{\gamma}\right) = \gamma \frac{w^2}{ML} . \qquad \square$$

We now start deriving an upper bound on the probability of error given in (1).

$$\begin{aligned} P[E] &= \Pr\{Y_j \geq Y_0, \text{ some } j \neq 0 | D = 0\} \\ &= \sum_{l} \Pr\{Y_j \geq Y_0, \text{ some } j \neq 0 | D = 0, \kappa_0^{M-1} = l_0^{M-1}\} \\ &\qquad \times \Pr\{\kappa_0^{M-1} = l_0^{M-1}\} \\ &\leq \sum_{l_1^{M-1}} \Pr\{Y_j \geq Y_0, \text{ some } j \neq 0 | D = 0, \\ &\qquad \kappa_0 = 0, \kappa_1^{M-1} = l_1^{M-1}\} \Pr\{\kappa_1^{M-1} = l_1^{M-1}\} . \end{aligned}$$

We further use a union bound, taking into account that the out-of-phase autocorrelation is bounded by one and that at most $\gamma - 1$ hits may occur in other slots due to code pulses in one slot of the desired user.

$$\begin{aligned} &\Pr\{Y_j \geq Y_0, \text{ some } j \neq 0 | D = 0, \kappa_0 = 0, \kappa_1^{M-1} = l_1^{M-1}\} \\ &\quad \leq (M - \gamma) \Pr\{Y_1 \geq Y_0 | D = 0, \kappa_0 = 0, \kappa_1 = l_1, \nu_1 = 0\} \\ &\qquad + \sum_{j=1}^{\gamma-1} \Pr\{Y_j \geq Y_0 | D = 0, \kappa_0 = 0, \kappa_j = l_j\} \\ &\quad \leq (M - \gamma) \Pr\{Y_1 \geq Y_0 | D = 0, \kappa_0 = 0, \kappa_1 = l_1, \nu_1 = 0\} \\ &\qquad + (\gamma - 1) q \Pr\{Y_1 \geq Y_0 | D = 0, \kappa_0 = 0, \kappa_1 = l_1, \nu_1 = 1\} \\ &\qquad + (\gamma - 1)(1 - q) \\ &\qquad\quad \times \Pr\{Y_1 \geq Y_0 | D = 0, \kappa_0 = 0, \kappa_1 = l_1, \nu_1 = 0\} , \end{aligned}$$

where $\nu_1 \in \{0, 1\}$ denotes the number of pulses that cause a hit in slot 1 due to the signature code pulses sent in slot 0 and $q = \Pr\{\nu_1 = 1\}$. Assuming uniformly distributed marks in the code sequences, we have $q = \frac{w(w-1)}{L-1}$. Whence

$$P[E] \leq \sum_{l} \left[(M - 1) P_1 + (\gamma - 1) \frac{w(w - 1)}{L - 1}(P_2 - P_1)\right] \Pr\{\kappa_1 = l\} \quad (2)$$

where

$$\Pr\{\kappa_1 = l\} = \binom{N-1}{l} p^l (1 - p)^{N-1-l} , \quad (3)$$

$$\begin{aligned} P_1 &= \Pr\{Y_1 \geq Y_0 | D = 0, \kappa_0 = 0, \kappa_1 = l, \nu_1 = 0\} \\ &= \sum_{y_1=0}^{\infty} e^{-Ql} \frac{(Ql)^{y_1}}{y_1!} \cdot \sum_{y_0=0}^{y_1} e^{-Qw} \frac{(Qw)^{y_0}}{y_0!} , \end{aligned} \quad (4)$$

and

$$\begin{aligned} P_2 &= \Pr\{Y_1 \geq Y_0 | D = 0, \kappa_0 = 0, \kappa_1 = l, \nu_1 = 1\} \\ &= \sum_{y_1=0}^{\infty} e^{-Q(l+1)} \frac{(Q(l+1))^{y_1}}{y_1!} \cdot \sum_{y_0=0}^{y_1} e^{-Qw} \frac{(Qw)^{y_0}}{y_0!} . \end{aligned} \quad (5)$$

Here Q denotes the average number of photons per transmitted pulse. Finally the bit error rate, P_b, can be obtained from the relation $P_b = \frac{M/2}{M-1} P[E]$.

IV. Numerical Results

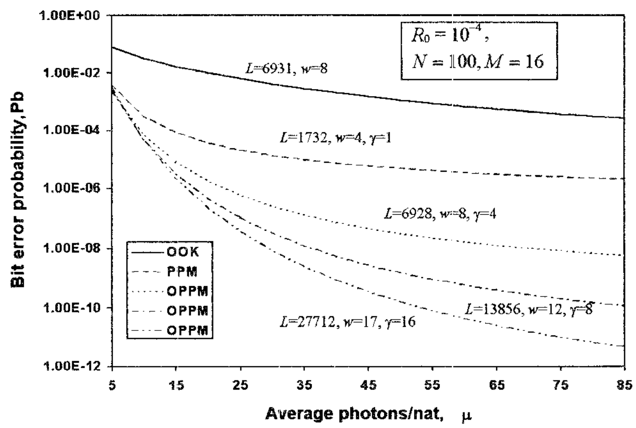

Fig. 6. A comparison between the bit error rates of optical OOK- and OPPM-CDMA receivers (with M=16 and different values of γ for OPPM) under the constraints of fixed throughput, chip time, and number of users.

In our numerical evaluations we assume that the rate of data transmission (throughput R_T) and pulsewidth, T_c, are both held fixed. Thus the product

$$R_0 \overset{\text{def}}{=} R_T T_c = \frac{\gamma \log M}{ML} \qquad \text{nats/chip}$$

is also fixed. Here the natural number e is taken as the basis of the "log" function.

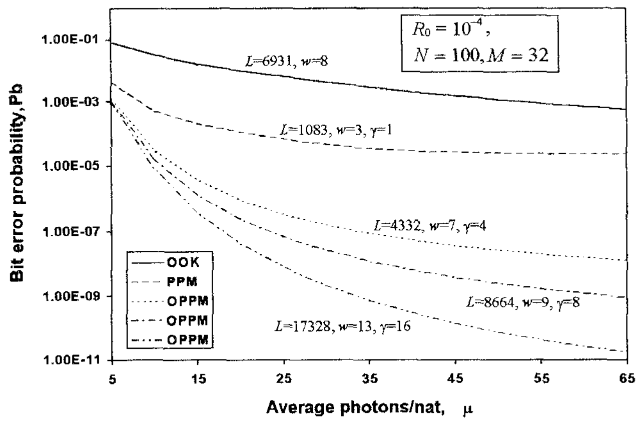

Fig. 7. A comparison between the bit error rates of optical OOK- and OPPM-CDMA receivers (with M=32 and different values of γ for OPPM) under the constraints of fixed throughput, chip time, and number of users.

We now examine the performance of the above system under a constraint on the average power (or energy per information nat). Let μ denote the number of transmitted photons per nat, Q in (4) and (5) is now equal to $Q = \frac{\mu \log M}{w}$. Figs. 6 and 7 show the bit error rate versus the average photons per nat for a fixed values of users and throughput-pulsewidth product. Different values of M and γ are assumed in both figures. The code length is chosen to satisfy the constraint on throughput. Maximum code weight is chosen so as to satisfy the constraint [16]:

$$N \leq \frac{L-1}{w(w-1)} .$$

The performance of OOK-CDMA is also superimposed on the same figures. The analysis of such a system can be found in [6] with slight modifications.

As seen from the figures, when γ increases, the system performance improves significantly. As an example, for a system with a 100 users, a 10^{-4} throughput-pulsewidth product, a 32 pulse-position multiplicity, and signal energy of 65 photons/nat, the bit error rates equal 5.62×10^{-4} for OOK-CDMA, 2.24×10^{-5} for PPM-CDMA, 1.24×10^{-8} for OPPM-CDMA with $\gamma = 4$, and 1.73×10^{-11} for OPPM-CDMA with $\gamma = 16$.

IV. Conclusion

Direct-detection optical CDMA systems with OPPM schemes have been studied. We considered optical orthogonal codes, with cross-correlations bounded by one, as the signature code sequences in our system. The Poisson shot noise model has been assumed for the receiver photodetector. The multiple-user interference has been accounted for in estimating the bit error rate. In our numerical evaluation we derived a union upper bound on the probability of error to simplify the calculations. We have evaluated the performance under the restriction of fixed throughput rate and chip time. We extract the following concluding remarks.

i) OPPM-CDMA system superperforms both OOK- and PPM-CDMA systems in terms of bit error rate.

ii) OPPM-CDMA system overcomes both the throughput limitation of PPM-CDMA system and the capacity limitation of OOK-CDMA system.

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