Design and Performance Analysis of a New Code for Spectral- Amplitude-Coding Optical CDMA Systems

Zhou Xiang, Member, IEEE, Hossam H. M. Shalaby, Senior Member, IEEE and Lu Chao, Member, IEEE
Nanyang Technological University,
School of EEE, Block S2, NTRC,
Nanyang Avenue, Singapore 639798
e-mail:ezhoux@ntu.edu.sg

***Abstract* – A new code structure for spectral amplitude optical code-division multiple-access is proposed. An upper bound on the code size is developed and an optimal construction that achieves this bound is presented. Further, a performance analysis of an optical CDMA system that utilizes this code is given. Our results reveal that such a new code can effectively suppress the intensity noise and in turn increase the number of the active users and improve the bit error rate performance.**

I. INTRODUCTION

One of the challenges of fiber optical networks is to design methods of supporting a large pool of subscribers, not all of them who require access to the network at the same time, while providing access to many simultaneously active users. Optical code division multiple access (OCDMA) is an attractive approach to meeting this challenges. The early OCDMA network proposals were based on coded sequences of incoherent pulses [1-2]. However, the used optical orthogonal codes (OOCs) generally have much poorer correlation properties than their bipolar counterparts, and the number of available codes is quite restricted. To overcome some of the limitation of these early proposals, a variety of coherent OCDMA systems have been developed [3-4]. Due to the coherent nature of these systems, they require some degree of matching or controlling of the light phase and polarization. More recently, several alternative OCDMA schemes based on spectral amplitude coding have been proposed [5-6]. These proposals both avoid the limitations of unipolar codes and do not require the complexity of coherent system. Unfortunately, their performance is limited by the intensity noise mainly originating from the interference between incoherent sources [6]. In this paper, we introduce a new code structure for spectral amplitude coding OCDMA systems. Our theoretical analysis shows that such a code structure can effectively compress the intensity noise and hence improve the bit error rate (BER) performance.

II. CODE DESIGN

For spectral encoding OCDMA systems, even with complete asynchronism between users, the frequency slots of different users will always be alighted. In this case, the multi-user interference can be thoroughly canceled theoretically as long as the used code ((0,1) sequences) satisfy the following properties: 1) all the codewords have the same weight w (defined as the number of "1"in it); 2) for every two different codewords $(X)=(x_1,x_2,\cdots x_n)$ and $(Y)=(y_1,y_2,\cdots y_n)$, we have $\Theta_{XY}=\sum_{i=1}^{N}x_i y_i=\lambda$, λ is a constant. Indeed any receiver that computes $\Theta_{XY}-[\lambda/(w-\lambda)]\Theta_{\bar{X}Y}$ will then reject the interference from any user having sequence (Y). Here we define $\Theta_{\bar{X}Y}=\sum_{i=1}^{N}(1-x_i)y_i=w-\lambda$. On the other hand, the encoder (the amplitude mask) is usually a passive device, and therefore some power loss will occur. Consequently, we need to make w as large as possible during the code construction. We define (n,w,λ) code as a family (0,1) sequences of length n, weight w and $\Theta_{XY}=\lambda$. In addition, the size of a (n,w,λ) code, denoted as $A(n,w,\lambda)$, is defined as the number of codewords in it. The conventional (0,1) m-sequences can be expressed as a $(N,(N+1)/2,(N+1)/4)$ code with size of N and Hadmard code as $(N,N/2,N/4)$ code with size of $N-1$. We can see that, the two codes have the same ratio of λ to w, i.e.1/2. In the next section, we show that, by appropriately decreasing the ratio of λ to w during code construction, the intensity noise can be reduced and therefore the allowable number of active users can be increased.

First, let us investigate the upper bound on the size of a (n,w,λ) code C. Let $B=(b_{ij})$ be an $M\times n$ array of the codewords of C (each row represents a codeword and M is the number of the codewords). The sum of the products of the rows is given by

$$S=\sum_{i=1}^{M}\sum_{\substack{j=1\\ j\neq i}}^{M}\sum_{v=1}^{n}b_{iv}b_{jv}=M(M-1)\lambda \tag{1}$$

On the other hand, if we use k_v as the number of 1's in the v^{th} column of B, the sum S is also equal to $\sum_{v=1}^{n}k_v(k_v-1)$. Note that $\sum_{v=1}^{n}k_v=wM$, and $\sum_{v=1}^{n}k_v^2$ is minimized if all $k_v=wM/n$, consequently we obtain $\frac{w^2M^2}{n} - wM \le \lambda M(M-1)$. Solving for M gives the upper bound as

$$A(n,w,\lambda) \le \frac{n(w-\lambda)}{w^2 - n\lambda} \quad \text{for} \quad w^2 > n\lambda \qquad (2)$$

Now we turn to the optimal construction of a (n,w,λ) code. Based on the theorem of block designs [7], we can obtain a symmetric $(\mathrm{v},b,r,\mathrm{k},\varsigma)$ block design by using the points and hyperplanes of the projective geometry $\mathrm{PG}(m,q)$, where $\mathrm{v} = b = \frac{q^{m+1}-1}{q-1}$, $r = \mathrm{k} = \frac{q^m - 1}{q-1}$ and $\varsigma = \frac{q^{m-1}-1}{q-1}$ (a $(\mathrm{v},b,r,\mathrm{k},\varsigma)$ block design consists of v objects and b blocks of these objects, with each object contained in r block, each block containing k objects, and each pair of objects contained in ς blocks; if v =b, then k also equal to ς, at this situation, the design is said to be symmetric.). The incidence matrix H of the symmetric design $(\mathrm{v},\mathrm{v},\mathrm{k},\mathrm{k},\varsigma)$ can then be given as

$$h_{ij} = \begin{cases} 1 & \text{if the } i\text{th block contains the } j\text{th element} \\ 0 & \text{otherwise} \end{cases} \qquad (3)$$

It is clear that H is a $\mathrm{v} \times \mathrm{v}$ matrix. For a symmetric block design $(\mathrm{v},\mathrm{v},\mathrm{k},\mathrm{k},\varsigma)$, it is known that every pair of blocks insect in the ς elements [7]. It is immediate that the rows of the matrix H construct a $\left(\frac{q^{m+1}-1}{q-1}, \frac{q^m-1}{q-1}, \frac{q^{m-1}-1}{q-1}\right)$ code with size of $\frac{q^{m+1}-1}{q-1}$. It is easy to see that the size of such a code design satisfies the upper bound of eq.(2), and the ratio of λ to w is tunable by choosing different values of q and m (by choosing q=3, m=4, we can construct a (121,40,13) code, if choosing q=4, m=4, we can construct another code (341,85,21)).

III. PERFORMANCE ANALYSIS

The ideal spectral-amplitude OCDMA system is shown in Fig. 1. The source spectra are assumed to be ideally flat over a bandwidth of $v_0 \pm \Delta v/2$, and ideal rectangular spectral-amplitude masking $A(v)$ is assumed as shown in Fig. 1. Unpolarized sources are assumed and each user is considered to have equal source power P_{sr}, more over, the transmission loss between any transmitter and receiver is assumed to be the same (denoted as γ hereafter). Note that the power loss due to encoding and the $1:\alpha$ coupler is not included in γ. These are best-case assumptions since we are interested in the upper bound on system performance.

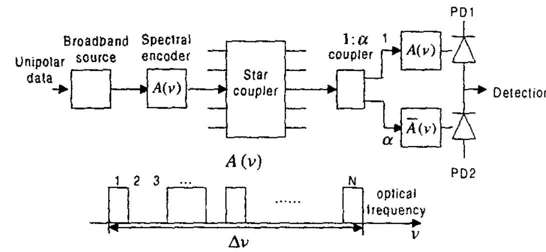

Fig. 1. Ideal spectral amplitude OCDMA system, $\alpha = \lambda/(w-\lambda)$

Due to the incoherent light from all users being incident on the receiver photodiodes, the interference effects of this light must be considered. According to the analysis in [6], if the field from many similar incoherent sources are mixed at a photodetector, the phase-induced intensity noise (PIIN) power will match that of a thermal source with the same power, even if the intensity noise from individual sources may be nonthermal. For the detection of light from a thermal source, assuming spatial coherence at the detector, the variance of the photodetector current can be expressed as [8]

$$< i^2 >= 2eIB + I^2(1+P^2)\tau_c B \qquad (4)$$

where e is the electronic charge, I is the average photocurrent, B is the noise-equivalent electrical bandwidth of the receiver, P is the degree of polarization of the source, and τ_c is coherence time of the source. In obtaining (4) it has been assumed that the optical bandwidth is much larger than the maximum electrical bandwidth. The source coherence time τ_c can be expressed in terms of its (one-sided) source power spectral density (PSD) $G(v)$ as [8]

$$\tau_c = \frac{\int_0^\infty G^2(v)dv}{\left[\int_0^\infty G(v)dv\right]^2} \qquad (5)$$

Let c_k ($c_k \in C$, C is a (N,w,λ) code) and d_k^l express the unipolar sequence used to amplitude code the source spectrum and the unipolar data of k th user, respectively. Without loss of generality, we assume that the receiver to have masks corresponding to codeword c_1. For total K active users, the power spectrum density (PSD) at the photodiodes PD1 and PD2 during one bit period can be expressed as

$$G_1(v) = \frac{\gamma P_{sr}}{\Delta v(1+\alpha)} \sum_{k=1}^{K} \sum_{i=1}^{N} \beta_k c_1(i) c_k(i) \left\{ u\left[v - v_0 - \frac{\Delta v}{2N}(-N+2i-2) \right] - u\left[v - v_0 - \frac{\Delta v}{2N}(-N+2i) \right] \right\} \quad (6)$$

$$G_2(v) = \frac{\gamma \alpha P_{sr}}{\Delta v(1+\alpha)} \sum_{k=1}^{K} \sum_{i=1}^{N} \beta_k [1 - c_1(i)] c_k(i) \left\{ u\left[v - v_0 - \frac{\Delta v}{2N}(-N+2i-2) \right] - u\left[v - v_0 - \frac{\Delta v}{2N}(-N+2i) \right] \right\} \quad (7)$$

respectively, where, $\alpha = \lambda/(w-\lambda)$, $u[v]$ and β_k is given as

$$u[v] = \begin{cases} 0 & \text{for } v < 0 \\ 1 & \text{for } v \geq 0 \end{cases} \quad (8)$$

$$\beta_k = \begin{cases} d_k^0(1-\tau_k) + d_k^{-1}\tau_k & \text{for } k \neq 1 \\ d_1^0 & k = 1 \end{cases} \quad (9)$$

In (9), τ_k means the normalized relative time delay between the user no. k and the desired user no.1. To obtain the knowledge of β_k, we consider two special cases, i.e. bit synchronous ($\tau_k \equiv 0$) and ideal bit asynchronous (τ_k distributes uniformly over [0,1]). Assuming that d_k^m take value 0 and 1 with an equal probability, the distribution density function of β_k ($k \neq 1$) can be given by

$$f(x) = \begin{cases} \begin{cases} \frac{1}{2} + \frac{1}{4}\delta(x) + \frac{1}{4}\delta(x-1) & 0 \leq x \leq 1 \\ 0 & \text{else} \end{cases} & \text{for bit asynchronous} \\ \frac{1}{2}\delta(x) + \frac{1}{2}\delta(x-1) & \text{for bit synchronous} \end{cases} \quad (10)$$

The photodiode current outputs from PD1 and PD2 are given by

$$I_1 = R.\int_0^\infty G_1(v) d = \frac{\gamma R P_{sr}}{N(1+\alpha)} \left[w\beta_1 + \lambda \sum_{k=2}^{K} \beta_k \right] \quad (11)$$

$$I_2 = R.\int_0^\infty G_{21}(v) d = \frac{\gamma R P_{sr}}{N(1+\alpha)} \left[\lambda \sum_{k=2}^{K} \beta_k \right] \quad (12)$$

respectively, where R denotes the photodiode responsivity. The signal from the desired user is then given as $I_1 - I_2$, and it is clear that the multi-user noise is completely cancelled. Note that the intensity noise satisfy Gaussian distribution, the intensity noise variance (conditioned upon β_k) from PD1 and PD2 can then be written as

$$\left\langle i_{in1}^2 \right\rangle_{\beta_k} = \frac{R^2 B (\gamma P_{sr})^2}{N \Delta v (1+\alpha)^2} \left\{ \beta_1^2 w + 2\lambda\beta_1 \sum_{k=2}^{K} \beta_k + \left(\lambda - \frac{w\lambda}{N} \right) \sum_{k=2}^{K} \beta_k^2 + \frac{w\lambda}{N} \sum_{k=2}^{K} \sum_{k=2}^{K} \beta_{k1}\beta_{k2} \right\} \quad (13)$$

and

$$\left\langle i_{in2}^2 \right\rangle_{\beta_k} = \frac{\alpha R^2 B (\gamma P_{sr})^2}{N \Delta v (1+\alpha)^2} \left\{ \left(\lambda - \frac{w\lambda}{N} \right) \sum_{k=2}^{K} \beta_k^2 + \frac{w\lambda}{N} \sum_{k=2}^{K} \sum_{k=2}^{K} \beta_{k1}\beta_{k2} \right\} \quad (14)$$

respectively, where

$$\varepsilon = \begin{cases} \frac{5}{12} & \text{for ideal asynchronous} \\ \frac{1}{2} & \text{for bit synchronous} \end{cases} \quad (15)$$

The total noise variance can be expressed as

$$< i_T^2 > = 2eB[< I_1 > + < I_2 >] + [< i_{in1}^2 > + < i_{in2}^2 >] + < i_{dark}^2 > + < i_{th}^2 > \quad (16)$$

where the terms represent shot noise, in tensity noise, dark current noise and thermal noise, respectively. In high power limit, the shot, dark and thermal noise can be neglected, giving the following signal to noise ratio (SNR):

$$\rho = \left[\frac{[I_1 - I_2]^2}{\left\langle i_{in}^2 \right\rangle_t} \right]_{\beta_1 = \frac{1}{2}} = \frac{\Delta v}{B\left\{ q_1 + (K-1)\left[\frac{2q_1}{q_2} + \frac{4\varepsilon\, q_1}{(q_2-1)} + \frac{(K-2)}{(q_2-1)} \right] \right\}} \quad (17)$$

where $q_1 = w/N$ and $q_2 = \lambda/w$. It is obvious that the SNR due to intensity noise limit can be improved by appropriate code design. For $\left(\frac{q^{m+1}-1}{q-1}, \frac{q^m-1}{q-1}, \frac{q^{m-1}-1}{q-1} \right)$ code as suggested in section II, if m is large enough, $q_1 = \frac{q^m - 1}{q^{m+1}-1} \approx \frac{1}{q}$ and $q_2 = \frac{q^{m-1}-1}{q^m - 1} \approx \frac{1}{q}$, then ρ can be reduced to

$$\rho \approx \frac{\Delta v}{B\left\{ q + (K-1)\left[2 + \frac{4.\varepsilon.q}{(q-1)} + \frac{(K-2)}{(q-1)} \right] \right\}} \quad (18)$$

From (18) we can see that, if K is large enough (approximately $(K-1)(K-2) > (q-1)^2 + 4\varepsilon(K-1)$), ρ will increase as q goes up, and therefore BER performance can be improved by choosing a relatively large value of q. This is clearly shown in Fig. 2, where we have taken $\Delta v = 20$ nm, $\varepsilon = 1/2$ and B =100 MHz (signal bit rate is 155 Mb/s). It can be observed that, for a BER of

10^{-9}, while q=2, 4 and 6, the allowable maximum number of active users is 25, 44 and 55, respectively. Apparently, the result for q=2 is just the same as that for conventional Hadmard code and m-sequences code.

In the above analysis, we only consider intensity noise; in fact, the value of q will also impose impacts on shot noise and thermal noise. When only shot noise is taken into account, the SNR can be given as $\frac{\gamma R P_{sr}(q-1)}{eB[4q+8(K-1)]q}$. It can be seen that, the SNR will reduce as q increases, but such an impact is definite only when K is relatively small ($K \approx q/2$) and is negligible for $K >> q/2$ and $(q-1)/q \approx 1$. Further, if only thermal noise is considered, the SNR is given as $\left[\frac{\gamma R P_{sr}(q-1)}{2q^2}\right]^2 \Big/ < i_{th}^2 >$. It can be observed that the SNR decreases rapidly as q increases (approximately in proportion to $1/q^2$). Consequently, we can conclude that the maximum allowable value of q is dependent on the impacts of the thermal noise.

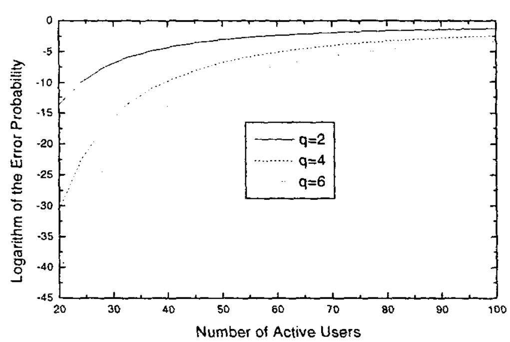

Fig. 2. Bit error rate as a function of K for various q using Gaussian noise approximation

Fig. 3 gives the calculated bit error rate as a function of effective source power (defined as γP_{sr}) for various values of q, where, we have taken thermal noise, shot noise and intensity noise into account. The used parameters are given as follow: K=50, ν_0=1550nm, $\Delta\nu = 20$ nm, $\varepsilon = 1/2$, B=100 MHz, $< i_{th}^2 >= 1.0\times10^{-17}$mA^2/Hz and photodiode quantum efficiency =0.8. It can be seen that, while γP_{sr} is relatively small, the bit error rate is much larger than that due to the intensity limit as shown in Fig. 2, and only when γP_{sr} is greater than a critical value (denoted as P_c hereafter), the intensity noise becomes dominate. It is worth-noting that the critical power value P_c is dependent on the value of q. The greater q is, the bigger P_c is. Typically, for q=2, 4, and 6, the corresponding P_c is around –27dBm, -22dBm and –18dBm. From above theoretical analysis we know that shot noise keeps almost invariant for various values of q if $K >> q/2$ and $(q-1)/q \approx 1$. It is then clear that thermal noise is the main reason that makes P_c increase along with q.

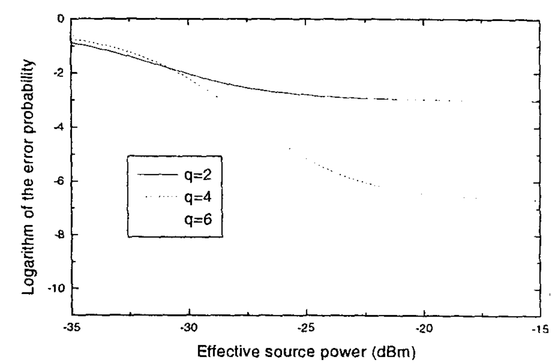

Fig. 3. Bit error rate as a function of effective source power γP_{sr} for various values of q

IV. CONCLUSIONS

We have proposed a new code structure for spectral amplitude coding in OCDMA systems. An upper bound on the code size has been developed and an optimal construction method is presented. From the performance analysis we conclude that the proposed code structure can effectively compress the intensity noise and improve the BER performance. Compared with the PPM scheme proposed in ref. [6], the method presented in this paper is simpler and effective.

REFERENCES

[1] J. Y. Hui, "Pattern code modulation and optical decoding-A novel code-division multiplexing technique for multi-fiber networks," *IEEE J. Selec. Areas Commun.*, vol. SAC-3, pp. 916-927, Nov. 1985.

[2] J. A. Salehi, "Code-division multiple-access techniques in optical fiber networks-part I: Fundamental principles," *IEEE Trans. Commu.* vol.37, pp. 824-833, Aug. 1989.

[3] A. M.Weiner, J. P. Heritage, and J. A. Salehi, "Encoding and decoding of femtosecond pulses," *Opt. Lett.*, vol.13, no.4, pp. 300-302, April 1988.

[4] M. E. Marhic and Y. L. Chang, "Pulse coding and coherent decoding in fiber–optic ladder networks," *Electronics Lett.*, vol.25, no.22, pp.1535-1536, Oct., 1989.

[5] M. Brandt-Pearce and B. Aazhang, "Optical spectral amplitude code division multiple access system," in Proc. *IEEE Int. Symp. Information Theory*, 1993, pp. 379.

[6] E. D. J. Smith, R. J. Blaikie, and D. P. Taylor, "Performance enhancement of spectral- amplitude-coding optical CDMA using pulse-position modulation," *IEEE Trans. Commu.* vol. 46, no. 9, pp. 1176 - 1185, Sep. 1998.

[7] I. Anderson, *Combinatorial Designs.* Chichester, England: Ellis Horwood Limited, 1990.

[8] J. W. Goodman, *Statistical Optics*. New York:Wiley, 1985.