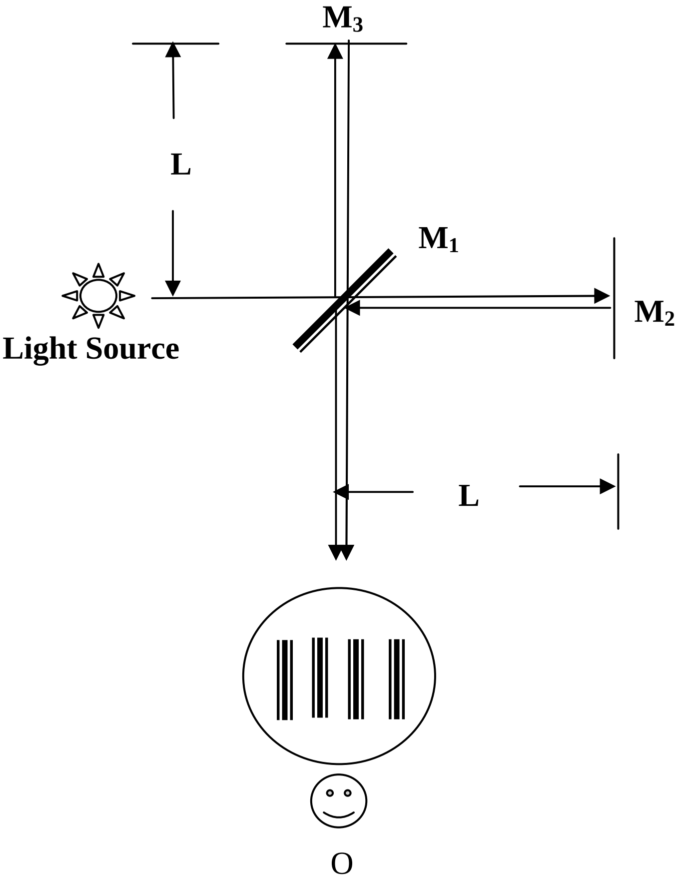
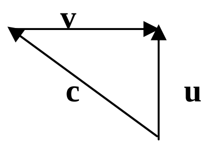
Lecture3

It was shown in the previous lecture that a correction factor of very small value $\cong 10^{-8}$ has been derived for the earth's motion. Such a very small value is hard to detect directly. This led Michelson and Morley to think of a set-up based of a difference measurement not on an absolute one, a device known as Michelson interferometer.





3.1 Michelson Interferometer

A collimated beam is split into two coherent beams by the half-silvered mirror M_1. One is transmitted to M_2 and one reflected to M_3. The two beams are reflected back from mirrors M_2 and M_3 to the mirror M_1. They get recombined and form an interference pattern, which is viewed and recorded by an observer O. For the transmitted beam the round trip time from M_1 to M_2 is written as:

$$t_{12} = \frac{2L}{c}(1+\frac{v^2}{c^2})$$

The perpendicular beam goes vertically to M_3 and comes back to M_1 by a velocity component $u = \sqrt{c^2 - v^2}$, hence the round trip time from M_1 to M_3 is written as:

$$t_{13} = \frac{2L}{\sqrt{c^2 - v^2}} = \frac{2L}{c}(1+\frac{1}{2}\frac{v^2}{c^2})$$

L is made equal between M_1 and each of M_2 and M_3. The time difference between the transmitted and reflected beam can then be written as:

$$\Delta t = t_{12} - t_{13} = \frac{2L}{c}(1 + \frac{v^2}{c^2} - 1 - \frac{1}{2}\frac{v^2}{c^2})$$

$$= \frac{2L}{c}\frac{1}{2}\frac{v^2}{c^2} = \frac{Lv^2}{c^2}$$

This time difference is to be detected by looking at the interference pattern of the two recombined beams of light. An argument was raised of how could the two L paths be mad exactly equal. The total apparatus is rotated 90^o to compensate for the path difference, i.e. the parallel beam becomes perpendicular and vice versa.

$$\Delta t` = -\frac{Lv^2}{c^3}$$

As a result of rotation, the phase reverses and becomes twice as much, the early beam now comes late and the late beam comes early.

$$\Delta t_{tot} = 2\frac{Lv^2}{c^3}$$

This time difference leads to a path difference $\Delta L = c\Delta t_{tot} = \frac{2Lv^2}{c^2}$, expressing this path difference in terms of the wavelength, the interference pattern should be shifted a number of fringes equal to the ratio $\Delta L / \lambda$.

$$\frac{\Delta L}{\lambda} = \frac{2L}{\lambda}\frac{v^2}{c^2} \text{ fringes}$$

Experimentally, Michelson interferometer has the following data:

L=1.2m, λ =590nm, and $\frac{v^2}{c^2} = 10^{-8}$, yielding:

$$\frac{\Delta L}{\lambda} = 0.04\, fringes\, .$$

Amazingly, there was no difference whatsoever observed. They carried out the experiment many times in different seasons of the year and at different places and the result was always ***negative.***

Conclusion: The speed of light does not depend on the relative motion of earth with through to the assumes ether, i.e. the speed of light is a ***constant*** .

3.2 The Special Theory of Relativity:

At this point in the course, we finally enter the twentieth century-- Albert Einstein wrote his first paper on relativity in 1905. To put his work in context, let us first review just what is meant by “relativity" in physics. The first example, mentioned in a previous lecture, is what is called "Galilean relativity" and is nothing but Galileo's perception that by observing the motion of objects, alive or dead, in a closed room there is no way to tell if the room is at rest or is in fact in a boat moving at a steady speed in a fixed direction. (You can tell if the room is accelerating or turning around). .Everything looks the same in a room in steady motion as it does in a room at rest. After Newton formulated his Laws of Motion, describing how bodies move in response to forces and so on ,physicists reformulated Galileo's observation in a slightly more technical, but equivalent, way: they said the laws of physics are the same in a uniformly moving room as they are in a room at rest. In other words, the same force produces the same acceleration, and an object experiencing no force moves at a steady speed in a straight line in either case. Of course, talking in these terms implies that we have clocks and rulers available so that we can actually time the motion of a body over a measured distance, so the physicist envisions the room in question to have calibrations along all the walls, so the position of anything can be measured, and a good clock to time motion. Such a suitably equipped room is called a "frame of reference"--the calibrations on the walls are seen as a frame which you can use to specify the precise position of an object at a given time. (This is the same as a set of "coordinates".) Anyway, the bottom line is that no amount of measuring of motions of objects in the "frame of reference" will tell you whether this is a frame at rest or one moving at a steady velocity .

What exactly do we mean by a frame "at rest" anyway? This seems obvious from our perspective as creatures who live on the surface of the earth--we mean, of course, at rest relative to fixed objects on the earth's surface. Actually, the earth's rotation means this isn't quite a fixed frame, and also the earth is moving in orbit at 18 miles per second. From an astronaut's point of view, then, a frame fixed relative to the sun might seem more reasonable. But why stop there? We believe the laws of physics are good throughout the universe. Let us consider somewhere in space far from the sun, even far from our galaxy. We would see galaxies in all directions, all moving in different ways. Suppose we now set up a frame of reference and check that Newton's laws still work. In particular, we check that the First Law holds--that a body experiencing no force moves at a steay speed in a straight line. This First law is often referred to as The Principle of Inertia, and a frame in which it holds is called an Inertial Frame. Then we set up another frame of reference, moving at a steady velocity relative to the first one, and find that Newton's laws are o.k. in this frame too. The point to notice here is that it is not at all obvious which--if either--of these frames is "at rest". We can ,however, assert that they are both inertial frames, after we've checked that in both of them, a body with no forces acting on it moves at a steady speed in a straight line (the speed could be zero). In this situation, Michelson would have said that a frame "at rest" is one at rest relative to the ether .However, his own experiment found motion through the ether to be undetectable, so how would we ever know we were in the right frame?

Now let us Generalize Galilean Relativity to Include Light and come to Einstein's major insight: the Theory of Special Relativity. It is deceptively simple.

There are two main postulates composing the special theory of relativity:

Postulate 1:

"The Fundamental laws of physics are the same in all Inertial Frames."

Of course this postulate implies the absence of a universal frame of reference. If the fundamental laws had different mathematical forms in different inertial frames, it would then be possible to tell which frame is stationary and which is in relative motion. This distinction can never be made because a frame at absolute rest does not exist. Therefore any observer can consider himself at rest and express all the fundamental laws of physics in a coordinate system which is at rest with respect to him. This coordinate system is then called a "Stationary System".

Postulate 2:

"The speed of light is a universal constant, it does not depend on relative motion between frames"

Derived from this postulate is that no energy can ever be transmitted aith a speed greater than that of light.

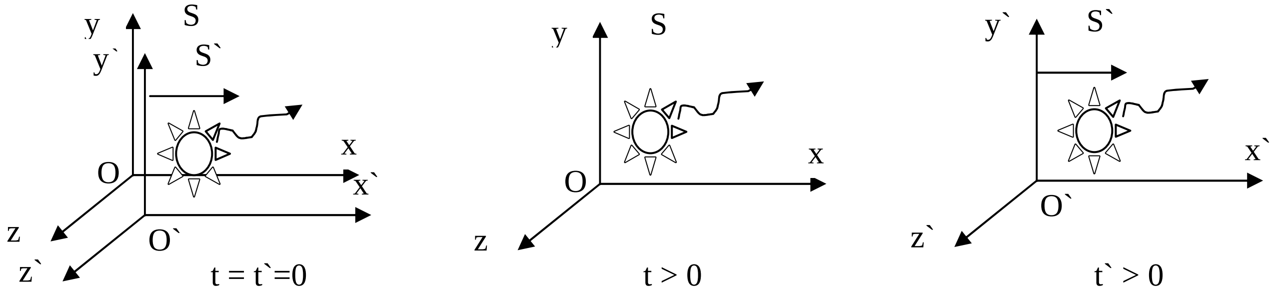
$$\mathbf{c = 3x10^8\ m/s}$$

<u>3.3 Lorentz Transformation:</u>

We will now derive a relativistic transformation providing that the following two assumptions are satisfied:

1. S and S\` are two inertial frames where S\` moves at a constant velocity v relative to S.
2. The speed of light is a constant and invariant either in S or S\`.

Assume a light pulse is transmitted at an instant t=t\`=0 when O coincides O\`, then we try to write the equation of the wave front in S and S\` and relate them together.



S: $x^2 + y^2 + z^2 = c^2t^2$ (1)

S\` $x^{`2} + y^{`2} + z^{`2} = c^2 t^{`2}$ (2)

Lorentz tried to find a transformation applied to (2) and results in (1). He assumed the following transformation:

$$\begin{aligned} x^` &= K(x - vt) \\ y^` &= y \\ z^` &= z \\ t^` &= At + Bx \end{aligned} \qquad (3)$$

where K, A , and B are constants to be determined by first substituting (3) into (2) and force the resultant equation to equal to (1).

$$K^2(x - vt)^2 + y^2 + z^2 = c^2(At + Bx)^2$$

$$K^2(x^2 - 2xvt + v^2r^2) + y^2 + z^2 = c^2(A^2t^2 + 2ABxt + B^2x^2)$$
$$x^2(K^2 - c^2B^2) + y^2 + z^2 = t^2(c^2A^2 - 2K^2v^2) + (2c^2AB + 2K^2v)xt \qquad (4)$$

Comparing now (4) to (1) results in:

$$K^2 - c^2B^2 = 1$$
$$c^2A^2 - K^2v^2 = c^2$$
$$c^2AB + K^2v = 0$$

Solving these three equation simultaneously gives:

$$K = A = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma > 1$$

$$B = -\frac{v}{c^2}\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

The transformation equations are then written as:

$$x` = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma(x - vt)$$
$$y` = y$$
$$z` = z$$
$$t` = \frac{t - vx/c^2}{\sqrt{1 - \frac{v^2}{c}}} = \gamma(t - vx/c^2)$$