Lecture 8

8.1 Quantum theory of light

Light of a given frequency is composed of individual photons, called quanta, whose energy is proportional to the frequency. Planck found that the energy contained in each photon is the same and is equal to:

$$E = h\upsilon \Rightarrow \textit{quantum energy}$$

8.2 Einstein's hypothesis

Light is emitted as separate quanta and travels in space in separate quanta too. When impinging on a metal surface, photoelectrons may be ejected and follows the following relation:

$$h\upsilon = K_{\max} + h\upsilon_o$$

$h\upsilon$: energy content in a quanta of the incident light.

$h\upsilon_o$: minimum energy to dislodge an electron from a metal surface being illuminated.

$k_{\max}$: maximum photo electron kinetic energy.

The minimum energy required to eject an electron from a metal surface is known as the work function φ.

Having defined the nature of light by quanta of energy, a burst of energy, light intensity can then be expressed as:

$$I = Nh\upsilon$$

where N is the number of incident photons per second per unit area. So we increase the light intensity I by increasing the number of incident photons N and not by the photon energy.

There are four main points as regard to the wave and quantum theory of light:

1. Light does not exhibit both its wave and particle nature at the same time.
2. The wave theory of light and the quantum theory complement each other. Each theory is only part of the whole story which is explained and completed by adding the other part. The true nature of light includes both wave and particle aspects.
3. Each theory is correct in a certain experiment and no relevant experiment that neither theory can account for.
4. We have no choice but to accept the dual nature of light, even though we can not visualize the true nature.

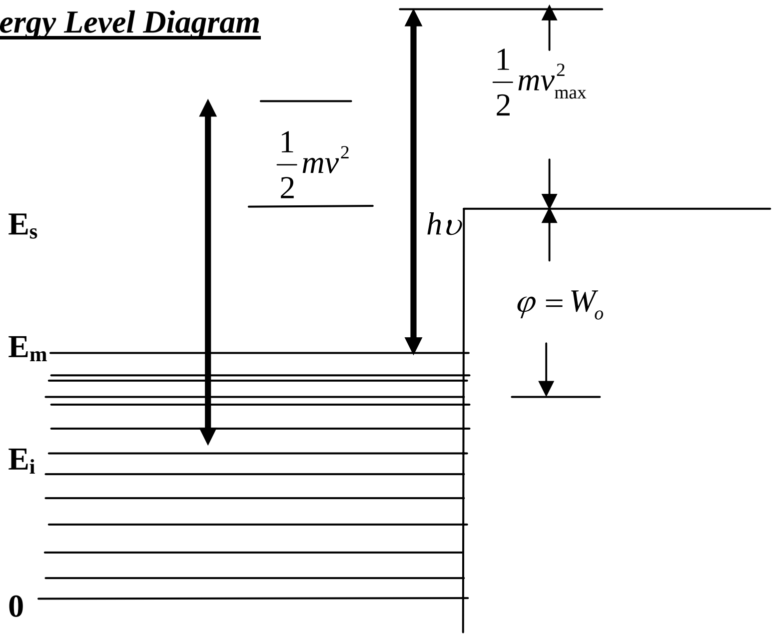
The least-tightly bound electron to a metal surface has to have an energy equal to the work function, φ, of the metal. If a photon, with higher energy, is absorbed by the least-tightly electron, it gains an additional amount of kinetic energy:

$$h\upsilon = \frac{1}{2}mv_{\max}^2 + \varphi$$

In general : $\frac{1}{2}mv^2 = h\upsilon - W$ for other electrons.

For least-tightly electrons: $W = W_o = \varphi$

8.3 Energy Level Diagram



$$h\upsilon + E_i = E_s + \frac{1}{2}mv^2$$

$$\therefore \frac{1}{2}mv^2 = h\upsilon - (E_i - E_s)$$

So, v is maximum if E_i is maximum ($E_i = E_m$):

$$\frac{1}{2}mv^2_{max} = h\upsilon - (E_s - E_m) = h\upsilon_o - W_o$$

$$\therefore \varphi = W_o = h\upsilon_o = E_s - E_m$$

8.4 Characteristics of a photon

A photon has no rest mass, m_o, photons are not found at rest , they are propagating as bundles of energy. One can now apply the relativistic energy equation:

$$m^2c^4 = E^2 = p^2c^2 + m_o^2c^4 = p^2c^2 + 0$$

$$\therefore E = pc$$

Photon as a wave (λ, υ)	Photon as a particle (m, p)
	$m = \frac{E}{c^2} = \frac{pc}{c^2} = \frac{p}{c}$
$E = h\upsilon$	$p = \frac{E}{c} = \frac{h\upsilon}{c} = \frac{h}{\lambda}$
	$E = pc$

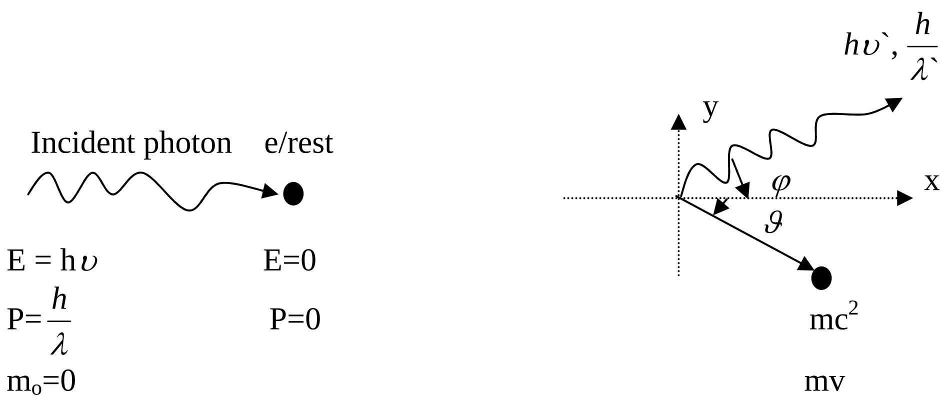
A wave can be totally described by a wave vector $\vec{K}$ whose direction is the direction of propagation, i. e. normal to the wave front, and whose magnitude is equal to $k = \frac{2\pi}{\lambda}$. The momentum of the photon can now be expressed as:

$$p = \frac{h}{\lambda} = \frac{2\pi h}{2\pi\lambda} = \frac{hk}{2\pi} = \hbar k$$

where $\hbar$ is equal to $\frac{h}{2\pi}$ and is called the reduced Planck's constant.

8.5 Compton Effect

In the experiment of the photoelectric effect, the total energy of the photon was assumed to be imparted to the electron. Let us try now to study a case where only a potion of the photon's energy is imparted to the electron as discussed by Compton. He assumed a collision between a photon and an electron as shown. The interaction of EM radiation with material particles is known as scattering.



Before collision **After Collision**

Now we apply the principles of conservation of momentum and the conservation of energy to this scattering phenomena.

Conservation of energy:

$$h\upsilon + m_o c^2 = h\upsilon` + mc^2$$

$$h\upsilon - h\upsilon` = m_o c^2(\gamma - 1) = Kinetic\ Energy$$

$$or\ \ mc^2 = h(\upsilon - \upsilon`) + m_o c^2 \qquad (1)$$

Conservation of momentum:

x-direction: $\frac{h}{\lambda} + 0 = \frac{h}{\lambda`}\cos\varphi + mv\cos\vartheta$ (2)

y-direction: $0 = \frac{h}{\lambda`}\sin\varphi - mv\sin\vartheta$ (3)

From (2): $mv\cos\vartheta = \frac{h}{\lambda} - \frac{h}{\lambda`}\cos\varphi$ (4)

From (3): $mv\sin\vartheta = \frac{h}{\lambda`}\sin\varphi$ (5)

Squaring (4) and (5) and add them together yields:

$$m^2v^2(\sin^2\vartheta + \cos^2\vartheta) = \frac{h^2}{\lambda^2} - \frac{2h^2}{\lambda\lambda`}\cos\varphi + \frac{h^2}{\lambda`^2}(\sin^2\varphi + \cos^2\varphi)$$

$$m^2v^2 = \frac{h^2}{\lambda^2} + \frac{h^2}{\lambda`^2} - \frac{2h^2}{\lambda\lambda`}\cos\varphi$$

$$p^2 = m^2v^2 = \frac{h^2}{\lambda^2\lambda`^2}(\lambda`^2 + \lambda^2 - 2\lambda\lambda`\cos\varphi) \qquad (6)$$

From (1): $mc^2 = hc(\frac{1}{\lambda} - \frac{1}{\lambda`}) + m_o c^2$

$$mc^2 = \frac{hc}{\lambda\lambda`}(\lambda` - \lambda) + m_o c^2$$

Squaring now the last equation gives:

$$E^2 = m^2c^4 = \frac{h^2c^2}{\lambda^2\lambda`^2}(\lambda`^2 - 2\lambda\lambda` + \lambda^2) + \frac{2m_o hc^3}{\lambda\lambda`}(\lambda` - \lambda) + m_o^2c^4 \qquad (7)$$

We have also the energy-momentum relation:

$$E^2 = p^2c^2 + m_o^2c^4 \qquad (8)$$

Substituting (6) and (7) into (8) gives:

$$\frac{h^2c^2}{\lambda^2\lambda`^2}(\lambda`^2 - 2\lambda\lambda` + \lambda^2) + \frac{2m_o hc^3}{\lambda\lambda`}(\lambda` - \lambda) + m_o^2c^4 =$$
$$\frac{h^2c^2}{\lambda^2\lambda`^2}(\lambda`^2 + \lambda^2 - 2\lambda\lambda`\cos\varphi) + m_o^2c^4$$

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$$\lambda` - \lambda = \frac{h}{m_o c}(1 - \cos\varphi)$$

$$\lambda` - \lambda = \lambda_c(1 - \cos\varphi)$$

where λ_c is equal to $\frac{h}{m_o c}$ and is known as Compton's wavelength.

$$\lambda_c = \frac{h}{m_o c} = \frac{6.626\times10^{-34}}{9.1\times10^{-31}\times3\times10^{8}} = 0.0242\,\overset{o}{A}$$

8.6 Remarks

1. The scattered wavelength $\lambda`$ is always larger than the incident wavelength λ since some of the incident energy has been imparted to the electron.
2. The difference in wavelength $\Delta\lambda = \lambda` - \lambda$ has an ultimate value of $2\lambda_c$ in case of head-on collision where the photon scatters opposite to the direction of incidence, i. e. $\varphi = 180^o$.
3. The difference in wavelength is independent of:

i- The incident wavelength.
ii- The nature of the scattering material.