Semiconductor Devices (EE336)

Lec. 6: Drift and Diffusion Currents

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Lecture Outline

- Carrier transport
- Thermal velocity of carriers
- Drift of carriers due to external applied electric field
- Mobility of charge carriers
- Diffusion of carriers due to carrier concentration gradient
- Diffusion constant
- Total current and its four components
- Einstein relationship between mobility and diffusion constant

Carrier transport

This part is concerned with motion of charge carriers (electrons and holes)

External applied electric field (Drift)

Changes in carrier concentration with position or its gradient (Diffusion)

For this part, I am closely following chapter 2 in this book

"Modern semiconductor devices for integrated circuits," by Chenming Hu Prentice Hall, 2010 [https://people.eecs.berkeley.edu/~hu/Book-Chapters-and-Lecture-Slides-download.html]

Thermal motion

- Even without applied Electric field, carriers are not at rest and possess finite kinetic energy due to thermal excitation

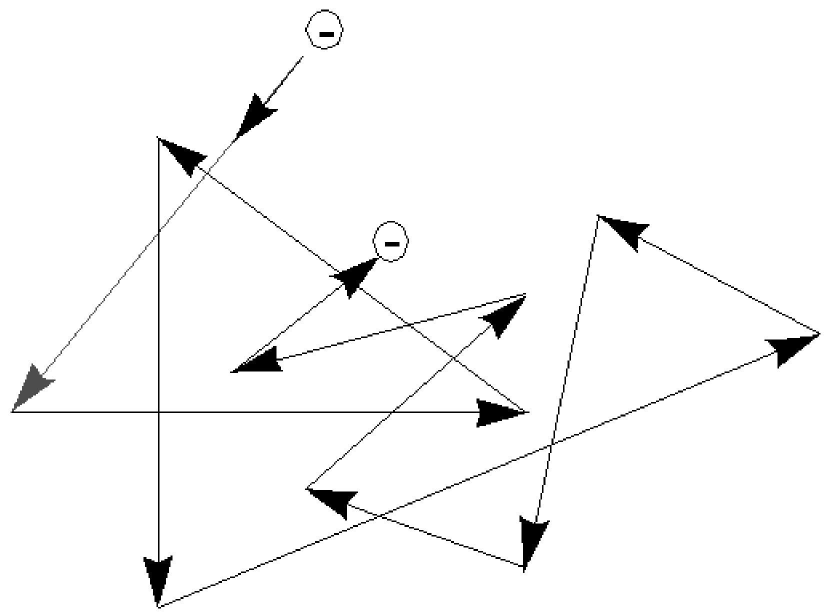
$$\text{Average electron K.E in CB} = \frac{\text{Total K.E.}}{\text{Elect. conc. in CB}} = \frac{\int_{E_c}^{\infty} f(E) N_c(E)(E - E_c)\, dE}{n_0}$$

$$\text{Average electron or hole kinetic energy } = \frac{3}{2}kT = \frac{1}{2}mv_{th}^2$$

$$v_{th} = \sqrt{\frac{3kT}{m_{eff}}} = \sqrt{\frac{3 \times 1.38 \times 10^{-23}\ \text{JK}^{-1} \times 300\ \text{K}}{0.26 \times 9.1 \times 10^{-31}\ \text{kg}}}$$

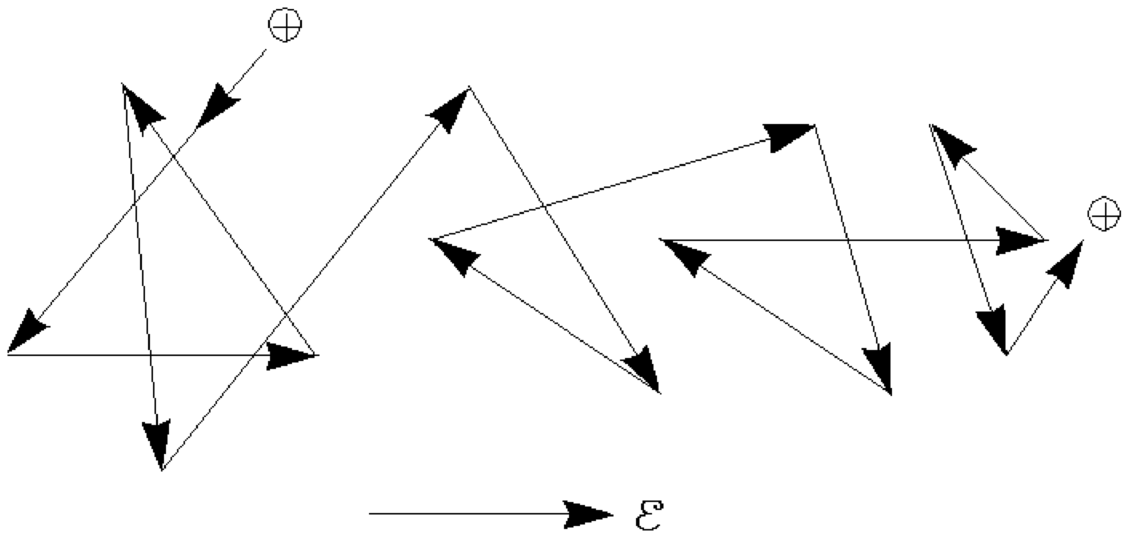
$$= 2.3 \times 10^5\ \text{m/s} = 2.3 \times 10^7\ \text{cm/s}$$

Thermal motion



- Zig-zag motion is due to collisions or scattering with imperfections in the crystal.
- Net thermal velocity is zero (averaged over many electrons at given time) and hence steady state current due to thermal motion is zero → only causes thermal noise
- Mean time between collisions is $\tau_m \sim 0.1\text{ps}$ (Mean free time)

Carrier drift



- ***Drift*** is the motion caused by an electric field.

Electron and hole mobility

$$m_p^* v_p = q\mathrm{E}\tau_{mp}$$

Momentum lost due to collision or scattering equals momentum gain between two scattering events due to external applied force (at steady state)

$$v_p = \frac{q\mathrm{E}\tau_{mp}}{m_p^*}$$

$$v_p = \mu_p \mathbf{E}$$

$$\mu_p = \frac{q\tau_{mp}}{m_p^*}$$

$$v_n = -\mu_n \mathbf{E}$$

$$\mu_n = \frac{q\tau_{mn}}{m_n^*}$$

- μ_p is the hole mobility and μ_n is the electron mobility
- τ_{mp} is the mean free time for holes and τ_{mn} is the mean free time for electrons which is the average time between two scattering events

Electron and hole mobility

$$v = \mu \mathbf{E}\ ; \quad \mu \text{ has the dimensions of } v/\mathbf{E} \quad \left[\frac{\text{cm/s}}{\text{V/cm}} = \frac{\text{cm}^2}{\text{V}\cdot\text{s}}\right].$$

Electron and hole mobilities of selected semiconductors

	Si	**Ge**	**GaAs**	**InAs**
μ_n (cm^2/V·s)	1400	3900	8500	30000
μ_p (cm^2/V·s)	470	1900	400	500

Based on the above table alone, which semiconductor and which carriers (electrons or holes) are attractive for applications in high-speed devices?

Numerical Example

Given $\mu_p = 470\ cm^2/V{\cdot}s$*, what is the hole drift velocity at* $\mathbf{E} = 10^3\ V/cm$*? What is* τ_{mp} *and what is the distance traveled between collisions (called the* ***mean free path****)? Hint: When in doubt, use the MKS system of units.*

Solution: $v = \mu_p \mathbf{E} = 470\ \text{cm}^2/\text{V}\cdot\text{s} \times 10^3\ \text{V/cm} = 4.7\times 10^5\ \text{cm/s}$

$$\tau_{mp} = \mu_p m_p / q = 470\ \text{cm}^2/\text{V}\cdot\text{s} \times 0.39 \times 9.1\times10^{-31}\ \text{kg}/1.6\times10^{-19}\ \text{C}$$

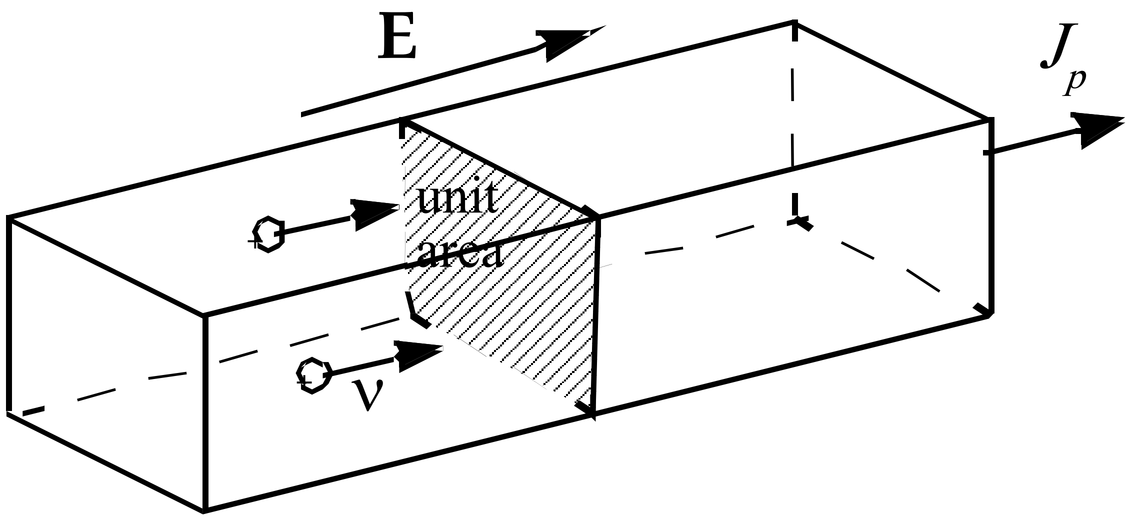
$$= 0.047\ \text{m}^2/\text{V}\cdot\text{s} \times 2.2\times10^{-12}\ \text{kg/C} = 1\times10^{-13}\text{s} = 0.1\ \text{ps}$$

$$\textit{mean free path} = \tau_{mh} v_{th} \sim 1\times 10^{-13}\ \text{s} \times 2.2\times10^{7}\ \text{cm/s}$$

$$= 2.2\times10^{-6}\ \text{cm} = 220\ \text{Å} = 22\ \text{nm}$$

This is smaller than the typical dimensions of devices, but getting close.

Drift current density



Hole current density $\boxed{J_p = qpv}$ A/cm^2 or C/cm^2·sec

EXAMPLE: If $p = 10^{15}$cm^{-3} and $v = 10^4$ cm/s, then

$$J_p = 1.6\times10^{-19}\text{C} \times 10^{15}\text{cm}^{-3} \times 10^4\text{cm/s}$$

$$= \; 1.6\,\text{C/s}\cdot\text{cm}^2 = 1.6\ \text{A/cm}^2$$

Drift current density

$$J_{p,drift} = qpv = qp\mu_p \mathbf{E}$$

Ohm's law

$$J_{n,drift} = -qnv = qn\mu_n \mathbf{E}$$

$$J_{drift} = J_{n,drift} + J_{p,drift} = \sigma \mathbf{E} = (qn\mu_n + qp\mu_p)\mathbf{E}$$

$\therefore$ **conductivity** (1/ohm-cm or S/cm) of a semiconductor is $\sigma = qn\mu_n + qp\mu_p$

$1/\sigma =$ is resistivity (ohm-cm)

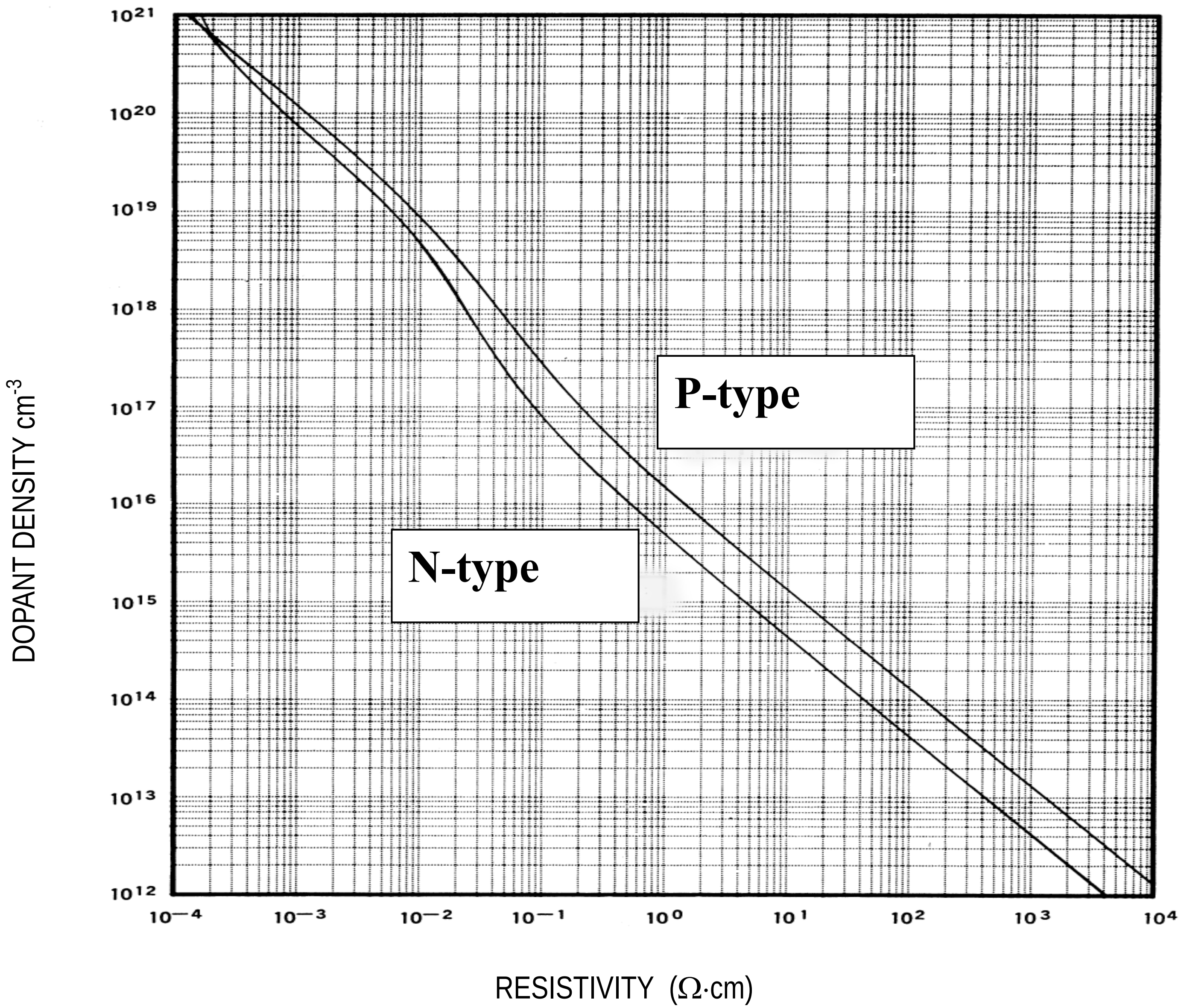
Numerical example

What is the resistivity of intrinsic Si? Use μ_n = 1350 and μ_p = 480 cm^2/V.s and n_i = 1.5×10^{10} cm^{-3}

$$\rho = \frac{1}{\sigma} = \frac{1}{q(\mu_n + \mu_p)n_i} = 2.28\times10^5 \quad \Omega.\text{cm}$$

- This number is expected to decrease when doping is made because of the increase in carrier concentration
- Be careful that the mobility will also decrease as the doping concentration increases due to larger impurity scattering as will be seen

Resistivity versus doping concentration for Si at room temp



Mechanisms of carrier scattering

There are two main causes of carrier scattering which impact carrier mobility:

1. Phonon Scattering (Phonon = lattice vibrations)
2. Ionized-Impurity (Coulombic) Scattering

Phonon scattering mobility decreases when temperature rises:

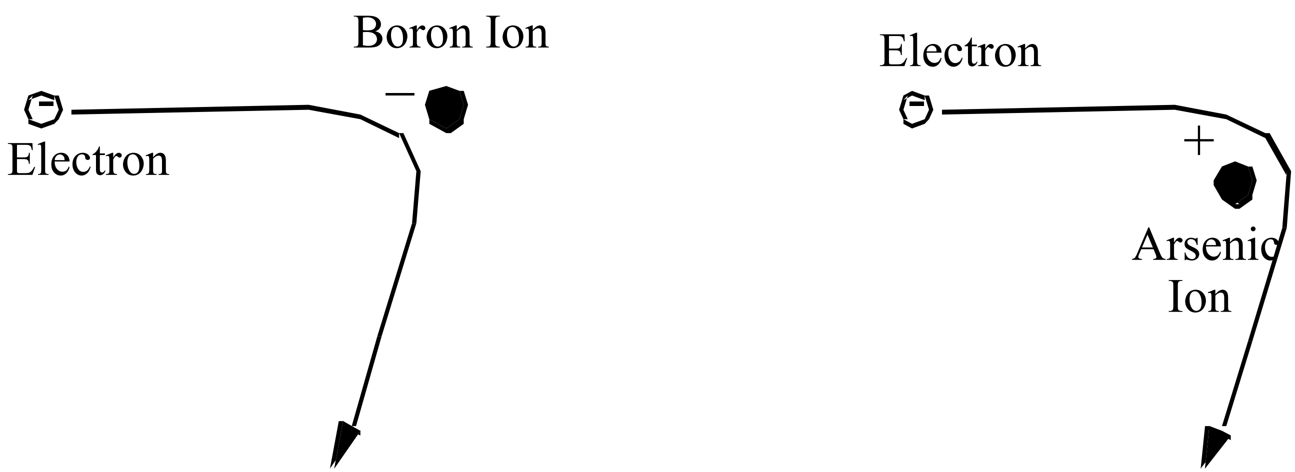
$$\mu_{phonon} \propto \tau_{phonon} \propto \frac{1}{phonon\ \ density \times carrier\ \ thermal\ \ velocity} \propto \frac{1}{T \times T^{1/2}} \propto T^{-3/2}$$

$\mu = q\tau/m$

$\propto T$

$v_{th} \propto T^{1/2}$

***Impurity (Dopant)-Ion Scattering* or Coulombic Scattering**



There is less change in the direction of travel if the electron zips by the ion at a higher speed.

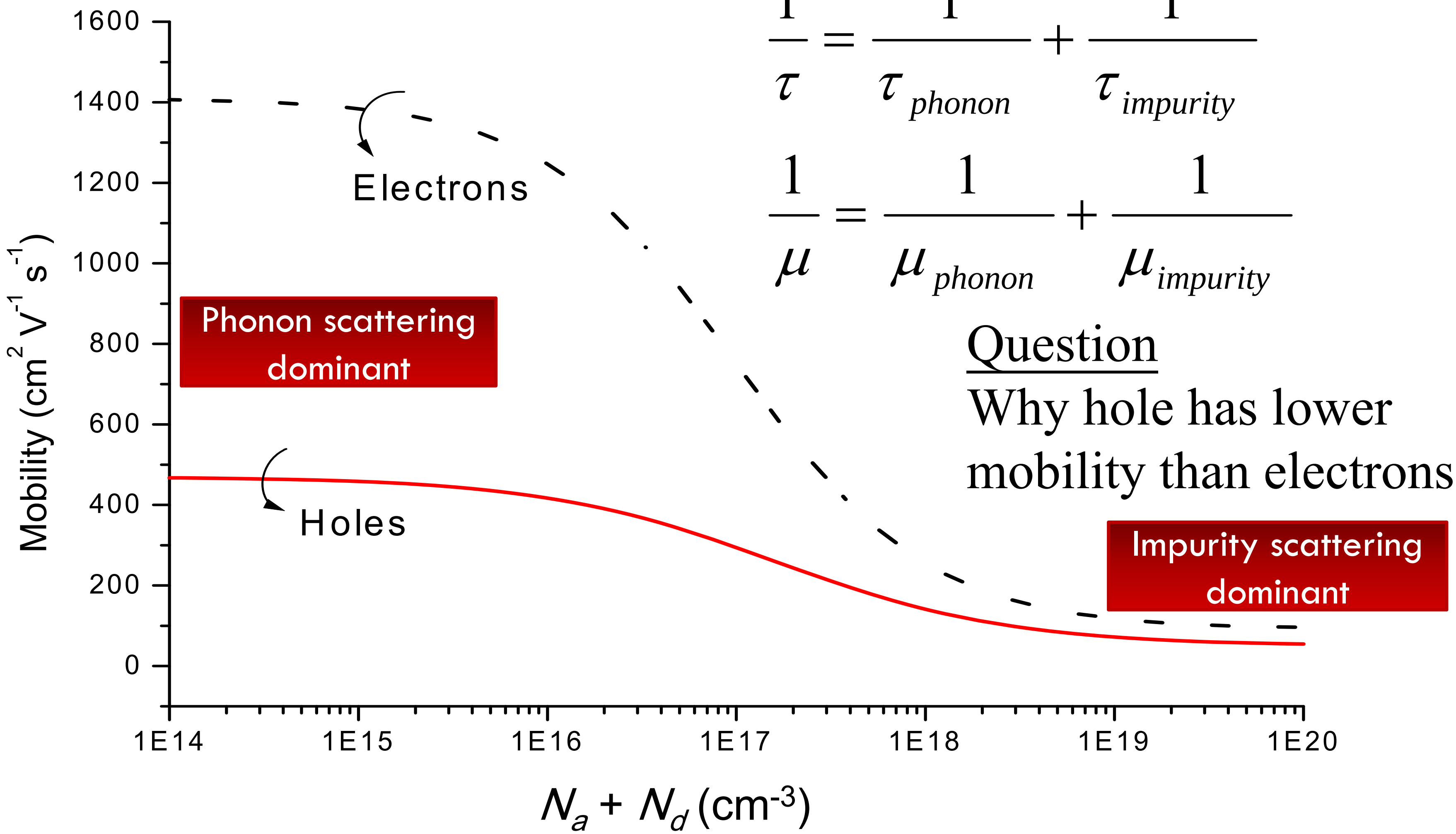
$$\mu_{impurity} \propto \frac{v_{th}^3}{N_a + N_d} \propto \frac{T^{3/2}}{N_a + N_d}$$

Mobility versus impurity concentration at fixed T = 300K

Total Mobility (sum of rates of two mechanisms)

$$\frac{1}{\tau} = \frac{1}{\tau_{phonon}} + \frac{1}{\tau_{impurity}}$$

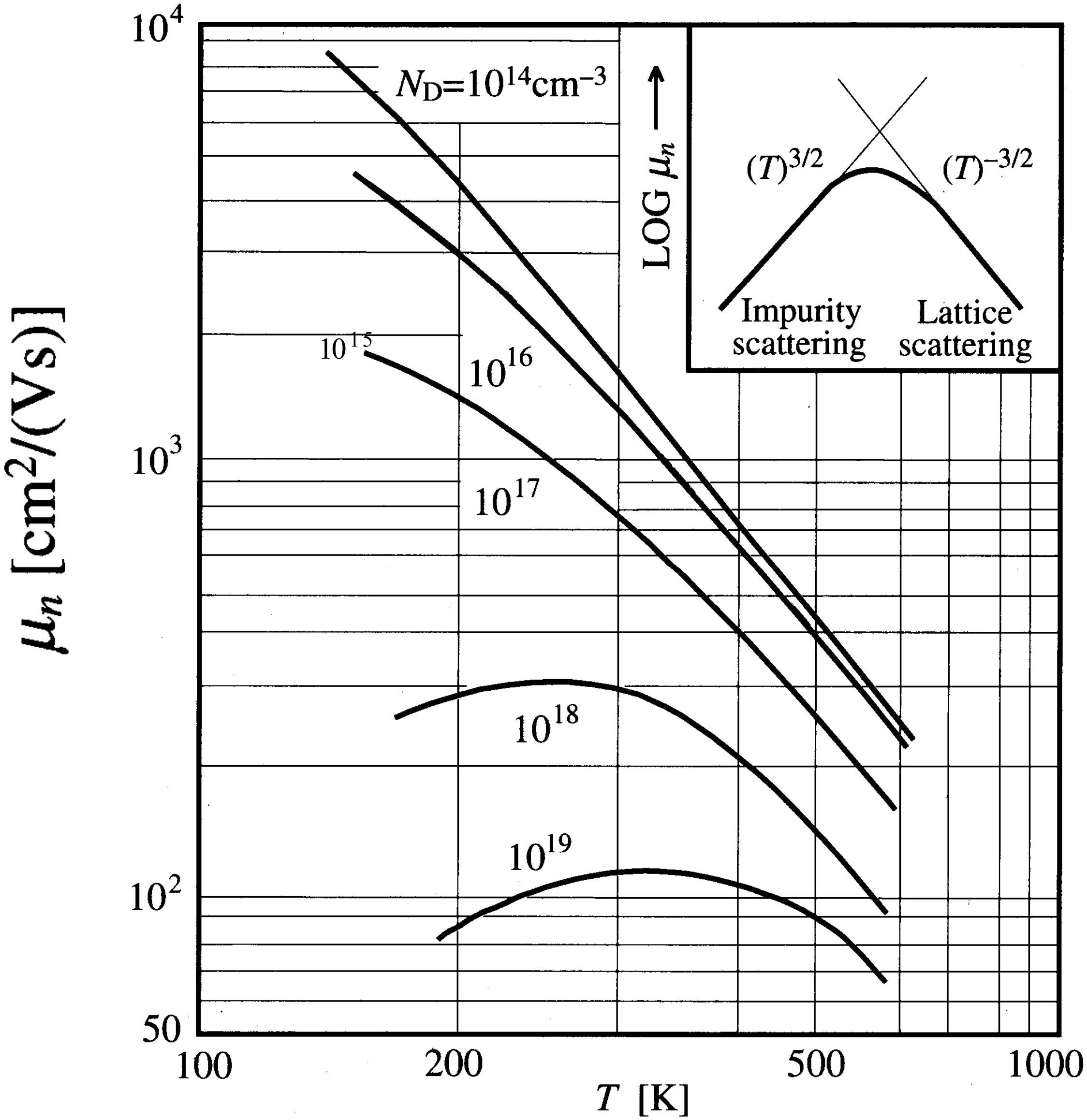
$$\frac{1}{\mu} = \frac{1}{\mu_{phonon}} + \frac{1}{\mu_{impurity}}$$



Question

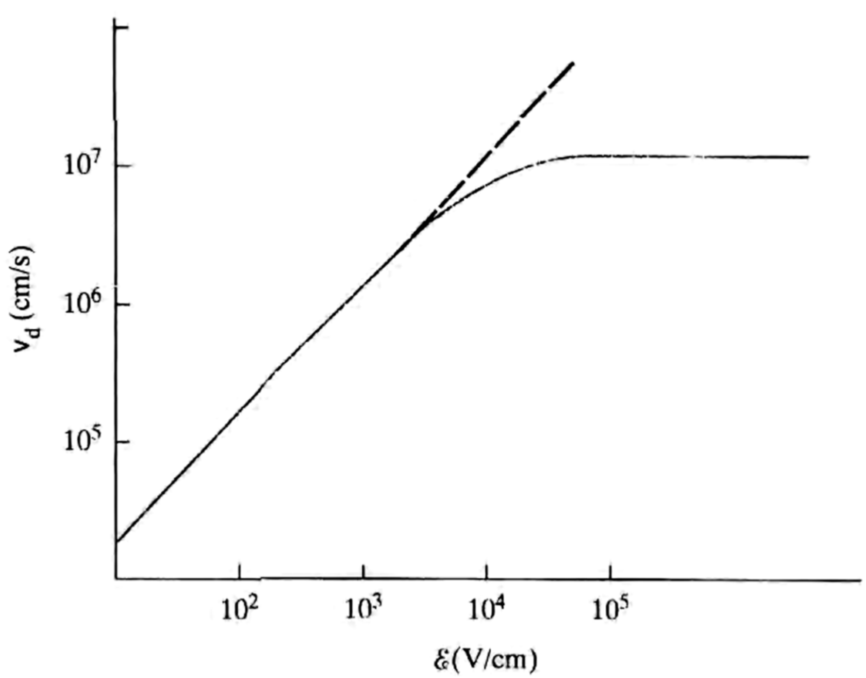
Why hole has lower mobility than electrons?

Temperature effect on mobility at various doping conc.

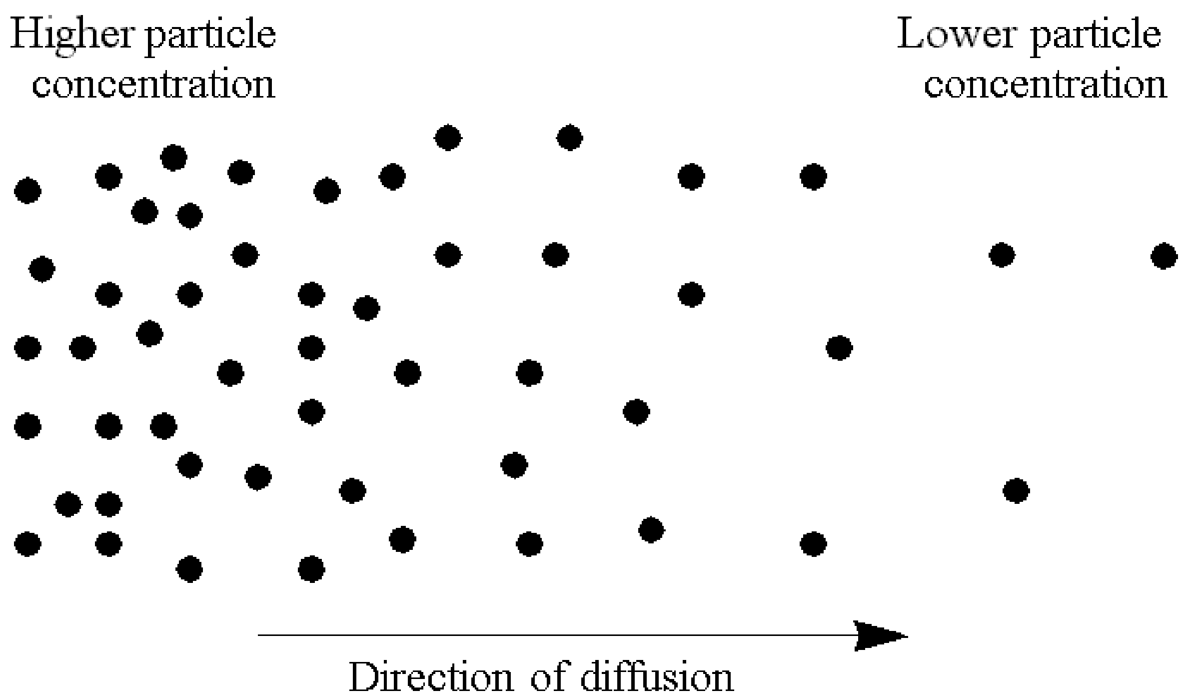


Velocity saturation (High field effects)

- When the kinetic energy of a carrier exceeds a critical value, it generates an optical phonon and loses the kinetic energy.
- Therefore, the kinetic energy is capped at large **E,** and the velocity does not rise above a saturation velocity, v_{sat} (scattering limited velocity) close to the thermal velocity of carriers
- ***Velocity saturation*** affects badly device speed



Diffusion of charge carrier

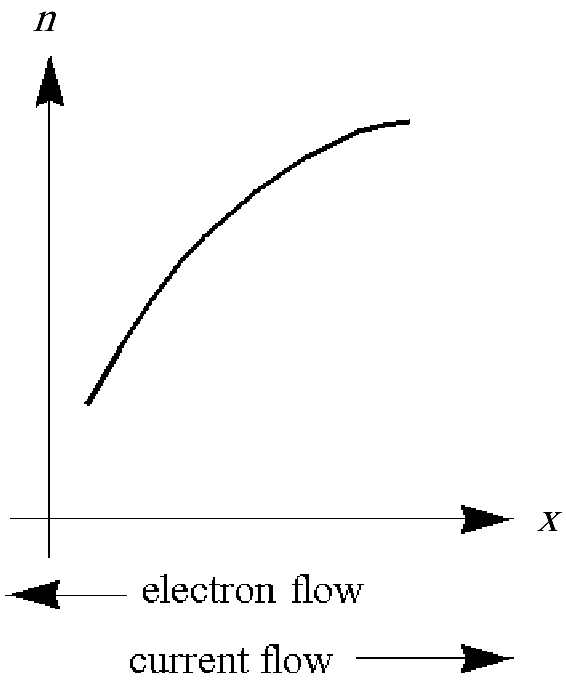


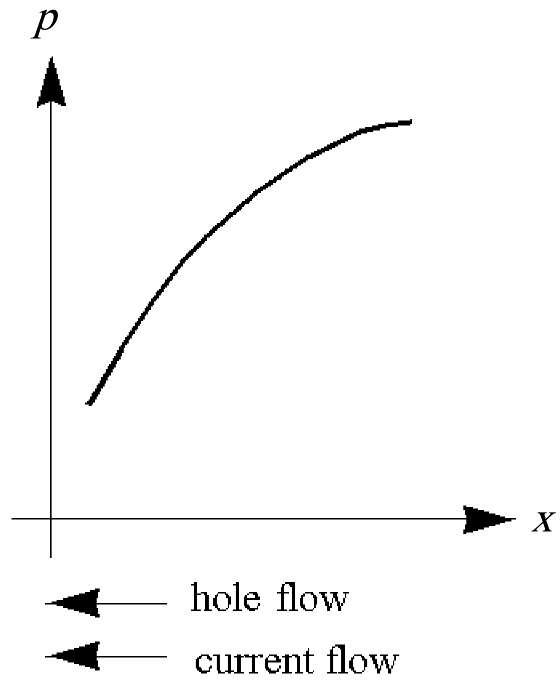
Particles diffuse from a higher-concentration location to a lower-concentration location → There must be concentration gradient for diffusion to occur (e.g. Non-uniform doping)

Diffusion current

$$J_{n,diffusion} = qD_n \frac{dn}{dx} \qquad J_{p,diffusion} = -qD_p \frac{dp}{dx}$$

D is called the diffusion constant. Signs explained:





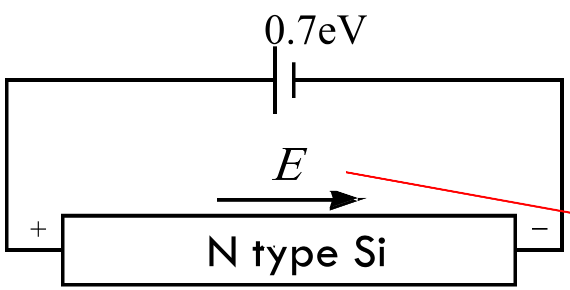
Total current (diffusion + drift)

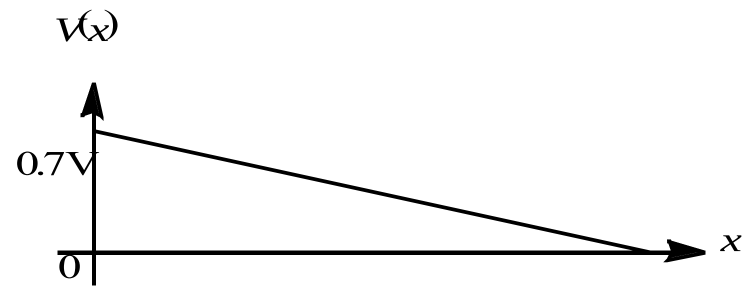
$$\boxed{J_{TOTAL} = J_n + J_p}$$

$$J_n = J_{n,drift} + J_{n,diffusion} = qn\mu_n\mathbf{E} + qD_n\frac{dn}{dx}$$

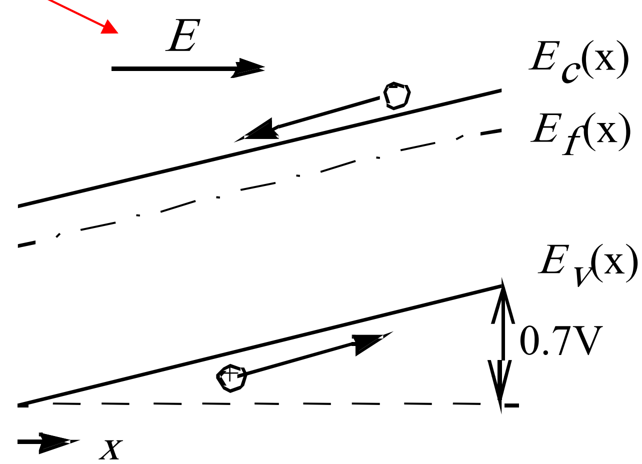
$$J_p = J_{p,drift} + J_{p,diffusion} = qp\mu_p\mathbf{E} - qD_p\frac{dp}{dx}$$

Relation between energy diagram and V & E





External applied
elect. field

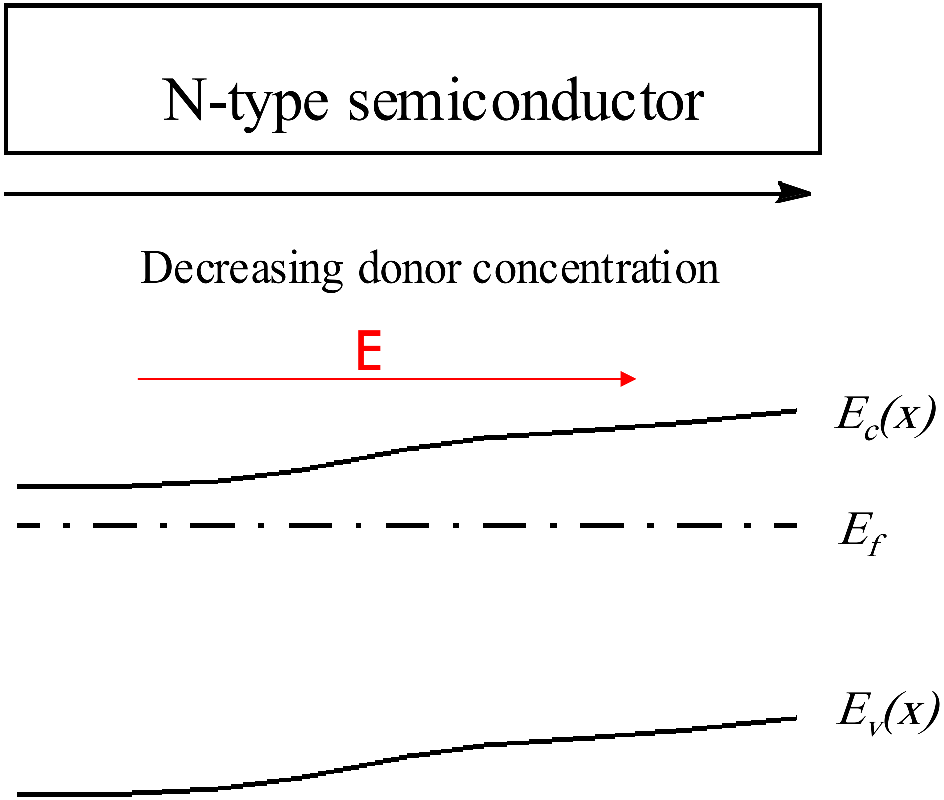


E_c and E_v vary in the opposite direction from the voltage. That is, E_c and E_v are higher where the voltage is lower.

$$E = -\frac{dV}{dx} = \frac{1}{q}\frac{dE_c}{dx} = \frac{1}{q}\frac{dE_v}{dx}$$

Einstein relationship between D and μ

Consider a piece of non-uniformly doped semiconductor at equilibrium (no external elect. Field and no net current flow).



$$n = N_c e^{-(E_c - E_f)/kT}$$

$$\frac{dn}{dx} = -\frac{N_c}{kT} e^{-(E_c - E_f)/kT} \frac{dE_c}{dx}$$

$$= -\frac{n}{kT}\frac{dE_c}{dx}$$

$$= -\frac{n}{kT} q\mathbf{E}$$

Internal electric field due to non-uniform doping that opposes the diffusion making net current zero

Is there an assumption that we made above about E_f and how it changes with x at equil. ??

Einstein relationship between D and μ

$$\frac{dn}{dx} = -\frac{n}{kT} q\mathbf{E}$$

$$J_n = qn\mu_n \mathbf{E} + qD_n \frac{dn}{dx} = 0$$

The net current at equilibrium must be zero (no external E)

$$0 = qn\mu_n \mathbf{E} - qn\frac{qD_n}{kT}\mathbf{E}$$

$$\boxed{D_n = \frac{kT}{q}\mu_n}$$

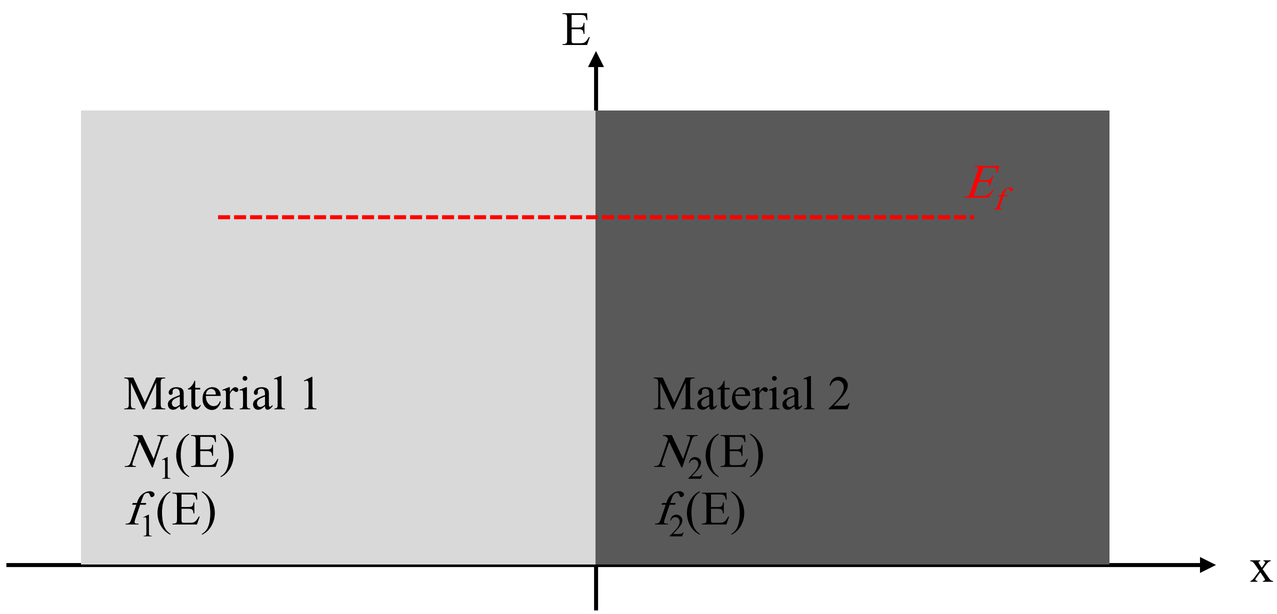
Similarly,

$$\boxed{D_p = \frac{kT}{q}\mu_p}$$

These are known as the Einstein relationship

	D_n (cm^2/s)	D_p (cm^2/s)	μ_n ($cm^2/V\text{-}s$)	μ_p ($cm^2/V\text{-}s$)
Ge	100	50	3900	1900
Si	35	12.5	1350	480
GaAs	220	10	8500	400

Invariance of Fermi level at equilibrium



- Two materials can for ex. be p-n junction, one SC non-uniformly doped, SC-metal junct.
- Since there is no net current flow at equilibrium the rate of flow of electrons from material 1 to 2 must be compensated by an equal flow rate of electrons from 2 to 1

$$\text{rate from 1 to 2} \propto N_1(E)\, f_1(E) \cdot N_2(E)\left[1 - f_2(E)\right]$$

$$\text{rate from 2 to 1} \propto N_2(E)\, f_2(E) \cdot N_1(E)\left[1 - f_1(E)\right]$$

$$f_1(E) = f_2(E) \quad \Rightarrow \quad E_{f_1} = E_{f_2}$$

$$\text{Generally at equilibrium} \quad \boxed{\frac{dE_f}{dx} = 0}$$

Example 1

What is the hole diffusion constant in a piece of silicon with $\mu_p = 410\ cm^2\ V^{-1}s^{-1}$?

Solution:

$$D_p = \left(\frac{kT}{q}\right)\mu_p = (26\,\mathrm{mV})\cdot 410\,\mathrm{cm^2V^{-1}s^{-1}} = 11\,\mathrm{cm^2/s}$$

Remember: kT/q = 26 mV at room temperature

Example 2

An intrinsic Si sample is doped with donors from one side such that $N_d(x) = N_0 e^{-ax}$.

(a) Find an expression for the built-in electric field E(x) at equilibrium over the range $N_d >> n_i$.

(b) Evaluate E(x) when a = 1 (μm)$^{-1}$.

(c) Sketch a band diagram and indicate the direction of E.

Solution:

a) $$E(x) = -\frac{kT}{q}\frac{dn/dx}{n} = -\frac{kT}{q}\frac{(-a)N_0 e^{-ax}}{N_0 e^{-ax}} = \frac{kT}{q}a$$

b) $$E(x) = \frac{kT}{q}a = 0.0259 \times 10^4 = 259 \ \text{V/cm}$$

c)