Faculty of Engineering
Alexandria University

BUCKLING OF SHIP STRUCTURE

Prof. Dr. M.A. Shama
Prof. of Naval Architecture & Shipbuilding
Dept. of Naval Architecture & Marine Engineering

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Prof. Dr. M. A. Shama

Prof. of Naval Architecture and Shipbuilding
Faculty of Engineering
Alexandria University, Egypt

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SI Units

International System of Units

This system can be divided into basic units and derived units as given in tables (1, 2).

Table (1): Basic Units

Quantity	Unit	Unit Symbol
Length	Meter	m
Mass	Kilogram	kg
Time	Second	s
Eclectic Current	Ampere	A
Thermodynamic Temperature	Degree Kelvin	°K
Luminous Intensity	Candela	cd

Table (2): Derived Units

Quantity	Unit	Unit Symbol
Force	Newton	$N = kg\ m/s^2$
Work, Energy	Joule	J = Nm
Power	Watt	W = J/s
Stress, Pressure	Pascal	$Pa = N/m^2$
Frequency	Hertz	H_z =1/s
Acceleration	Meter/second squared	$g = m/s^2$
Area	Square meter	m^2
Volume	Cubic Meter	m^3
Density	Kilogram/c	$\gamma = kg/m^3$= tonne/m^3
Velocity	Meter/second	υ= m/s
Angular velocity	Radian/second	Rad/s
Dynamic Viscosity	Newton second/meter sq.	Ns/m^2
Kinematic Viscosity	Meter squared/second	M^2/s
Thermal Conductivity	Watt/meter degree Kelvin	W/(m°k)

Table (3). Conversion Tables

A summary of the quantities commonly used in naval architecture is given in the following table together with the corresponding SI units.

Quantity	Common Unit	SI Unit
Length	1 ft	0.3048 m
	1 nautical mile	1,842 m
	1 mile	1,609 m
Area	1 ft^2	0.0929 m^2
Volume	1 ft^3	0.02832 m^3
Velocity	1 ft/s	0.3048 m/s
	1 knot	0.5144 m/s
Standard Acceleration	32.174 ft/s^2	9.8066 m/s^2
Mass	1 Ib	0.4536 kg
	1 ton	1,016 kg
	1 ton	1.016 tonne
Force	1 1bF	4.4482 N
Pressure, Stress	1 $1b/in^2$	6.8947 KN/m^2
	1 t/in^2	15.444 MN/m^2
Energy	1 ft 1b	1.3558 J
Power	1 hp	745.7 W
Density (SW)	35 ft^3/ton	0.975 m^3/tonne
	1.025 tonne/m^3	0.01 MN/m^3
Density (FW)	36 ft^3/ton	1.0 m^3/tonne
	1.0 tonne/m^3	0.0098 MN/m^3
Modulus of Elasticity	13,500 tons/in^2	E = 20.9 GN
TPI (SW)	A_W/420 tonf/in	1.025A_w tonne/m
TPM	100.62 A_W (N/cm)	10^4 A_W (N/m)
MCT" (sw)	$\frac{\Delta \overline{GM}_L}{12L} \frac{tonf \cdot ft}{in}$	$\frac{\Delta \overline{GM}_L}{L} \left(\frac{MN_m}{m} \right)$
Displacement Force	1 ton f	1.016 tonnef
Wetted surface	$S=2.28\sqrt{\Delta L}$	Δ = tones, L=m

Table (4) Power conversion

Quantity	Common Unit	SI Unit
BHP	P_B	W
SHP	P_S	W
DHP	P_D	W
EHP	P_E	W

Table (5): Multiples and sub – multiples.

Prefix	Factor	Symbol
Tera	10^{12}	T
Giga	10^9	G
Mega	10^6	M
Kilo	10^3	k
Milli	10^{-3}	m
Micro	10^{-6}	µ
Nano	10^{-9}	n
Pico	10^{-12}	p
Femto	10^{-15}	f
Atto	10^{-18}	a

General units

Gravity acceleration g = 9.807 m/s^2
Water density (salt water) γ_{sw} = 1.025 t/m^3
Modulus of elasticity E = 20.9 MN/cm^2
Atmospheric pressure = 10.14 NKm^2
1.0 ton displacement = 9964 N

Shipbuilding Units

(a) General:

• dimensions/distances	m
• primary spacings	m
• secondary spacings	mm
• area	m2
• volume	m2
• mass	kg
• velocity	m/s
• acceleration	m/s^2

(b) Hull girder properties:

• dimensions	m
• area	m^2
• section modulus	m^3
• inertia	m^4
• moment of area	m^3

• dimensions	mm
• area	cm^2
• section modulus	cm^3
• inertia	cm^4
• length/effective length	m

(d) Plating dimensions:

• breadth	mm
• length	m
• thickness	mm

(e) Loads:

• pressures	kN/m^2
• loads	kN
• bending moment	kNm
• shear force	kN

(f) Miscellaneous:

• yield strength	N/mm^2
• stress	N/mm^2
• deflections	mm
• modulus of elasticity	N/mm^2
• weight	tonnes
• density	tonnes/m^3
• displacement	tonnes
• angle	deg
• calculated angle	rad
• period	s
• frequency	Hz
• ship speed	knots

Buckling of ship structure

Introduction

Buckling or structural instability is considered one of the main modes of failure of ship structural elements.

The stability phenomenon of ship structures is defined by the state of equilibrium of structural members.

The equilibrium of the designed structure is stable if small imperfections and defects will cause correspondingly small deviations from the idealized operating conditions. If small imperfections cause disproportionately large deviations, the equilibrium is unstable.

All idealized structure differs from the actual structure due to small deviations, defects and imperfections. Despite the presence of these deviations, the actual structure should operate and perform in a manner similar to its corresponding idealized structure. Therefore, the size of structure (geometry and scantlings) should be selected to ensure that stable equilibrium would occur under all kinds of perturbations. Thus, the stability concept describes the relationship between the perturbing causes and the resulting consequences.

The equilibrium of the system is called stable if the system returns to the initial equilibrium state after the removal of a small disturbing force.

Let u_i, i= 1, 2, 3,…m = reasons causing a deviation from the unperturbed equilibrium state

v_j, j = 1, 2, 3, ..n = resulting consequences

The unperturbed equilibrium is called stable, over a time interval T, when:

$|u_i| \leq \varepsilon_i$

Where ε_I = selected small positive number that would give:

$|v_i| < \eta_j$

Where η_j = small positive number

The most important classes of perturbations are the initial curvature, or lateral deflection, and the eccentricity of the compressive force.

The perturbations common in practice are not completely deterministic but are subject to statistical variations.

In general, initial lateral deflection δo = L/750

Eccentricity e = 0.1r, r core radius = Z/A

Column Buckling

Boundary conditions of columns

The boundary conditions of columns could be either free, hinged, fixed or constrained, as shown in fig.(1).

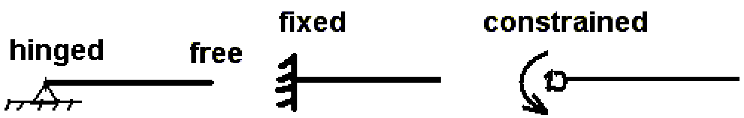


Fig. (1). Boundary support conditions

Generalized Euler Formula

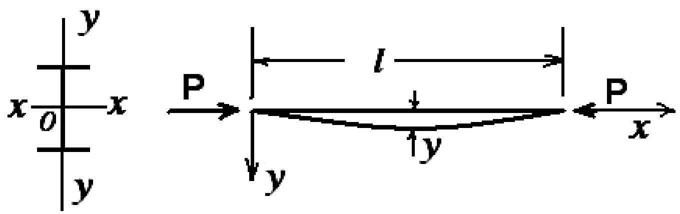


Fig. (2). Idealized beam column

The generalized Euler formula for a column subjected to end compressive force P, see Fig. (2), is given by:

$$= \frac{\quad}{L_e}$$

= **Euler buckling stress and is given by:** $= \frac{\quad}{\quad}$

Hence: $= \frac{\quad}{L_e}$

Where: σ_E **= Euler buckling stress**

P_E **= Euler buckling load**

P = Compressive force on column

A = sectional area of column

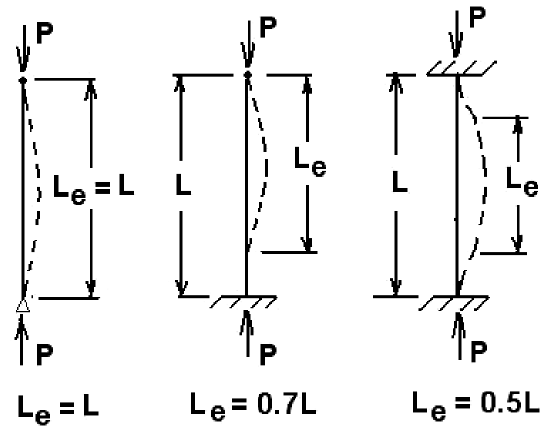


Fig. (3). Effective length of columns

Let: $r^2 = I/A$ **and** $\lambda = \frac{L_e}{\quad}$

Where:

r = radius of gyration

$L_e = L_e = kL$ **= Effective length of column**

k = a factor depending on the type of end supports, see Fig.(3).

k = 1.0 for hinged supports

k = 0.5 for fixed end supports

k = 0.7 hinged-fixed supports

λ **=slenderness ratio of the column**

For elastic buckling, the limiting value of λ is given by:

$$\lambda = \sqrt{\frac{}{\sigma_{pr}}}$$

Where: σ_{pr} = proportional limit of the material

For shipbuilding steel, we have:

$E = 21 \times 10^5$ kg/cm^2 and $\sigma_{pr} = 2000$ kg/cm^2

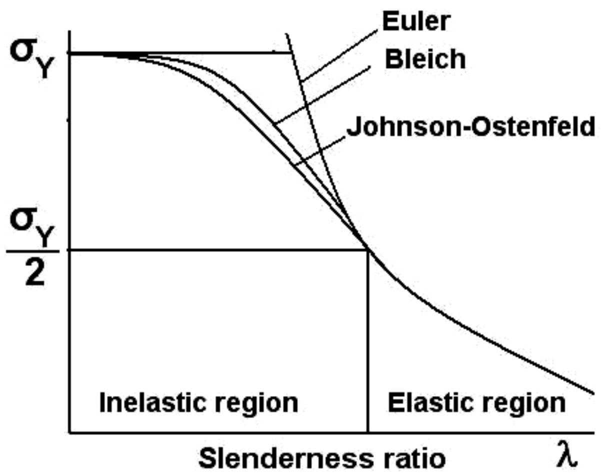


Fig.(4). Euler buckling curve

Hence: the limiting value of λ for elastic buckling of columns is given by, see Fig.(4):

$$\lambda = \sqrt{\frac{}{\sigma_{pr}}} = 3.14\sqrt{\frac{}{2000}} = 105$$

For $\sigma_E \le 0.5\sigma_y$, $\sigma_{cr} = \sigma_E$, see Fig.(4)

For $\sigma_E \ge 0.5\sigma_y$ $\quad \sigma_{cr} = \sigma_y\left[1 - \frac{\sigma_y}{4\sigma_E}\right]$

Where: σ_y = lower yield stress

σ_{cr} = critical buckling stress

Example - Calculate the critical buckling stress for a column having the following data for the two cases:

1- The column is fixed at both ends

2- The column is hinged at both ends

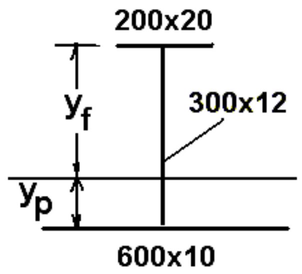
Given: A = 136 cm^2,

I = 24538.2 cm^4

y_f = 17.206 cm, y_P = 12.794

L = 3.0 m, E = 2.1 $\times 10^7$ t/m^2

$= \quad /$



1-<u>The column is fixed at both ends:</u>

Then: $L_e = - =$ m and $\pi^2 EI / L_e^2$

$$= \pi^2 \times 2.1 \times 10^7 \times 24538.2 \times 10^{-4} / 1.5^2 \times 100^2 = 22580.77 \text{ t}$$

$$= \pi^2 \times 2.1 \times 24538.2 \times 10^{-1} / 1.5^2 = 22580.77 \text{ tons}$$

$\sigma_e = \frac{22580.77}{136} = 166.035$ t/cm^2, hence: $\sigma_e > \frac{\sigma_y}{2}$

Then: $\sigma_{er} = \sigma_y\left[1 - \frac{\sigma_y}{4\sigma_e}\right] = 2.4\left[1 - \frac{2.4}{4 \times 166.035}\right] = 2.391$ t/cm^2

2-The column is hinged at both ends

Then $L_e = \quad = \overline{\qquad} =$

$\sigma_e = \frac{5645.2}{136} = 41.51 \ \text{t/cm}^2$, **hence: $\sigma_e > \sigma_y/2$**

$$\sigma_{cr} = 2.4\left[1 - \frac{2.4}{4 \times 41.51}\right] = 2.365 \ \text{t/cm}^2$$

Rational Shapes of Column Sections in Compression

The radius of gyration $k = \sqrt{\frac{I}{A}}$

Let : $\rho = \frac{k}{\sqrt{A}} = \frac{\sqrt{I/A}}{\sqrt{A}} = \frac{\sqrt{I}}{A}$

Where: ρ = unit radius of gyration (non –dimensional)

Efficient sections have high ω values, as give in the following table:

Section			ρ	Note
Rectangle		h/b=2	0.204	Least economy
Square			0.289	
Circle			0.283	
Ring		r/R=0.8-0.95	1.6-2.25	Most economical
I-section			0.27-0.41	
Angle			0.3-0.5	

Beam columns

Basic configurations and loadings

The basic configuration and loading of a beam column in the bottom structure of a ship hull girder is shown in figs.(5,6).

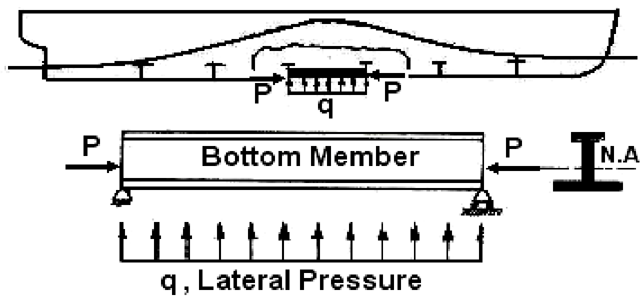


Fig. (5).A beam column of a ship bottom structure

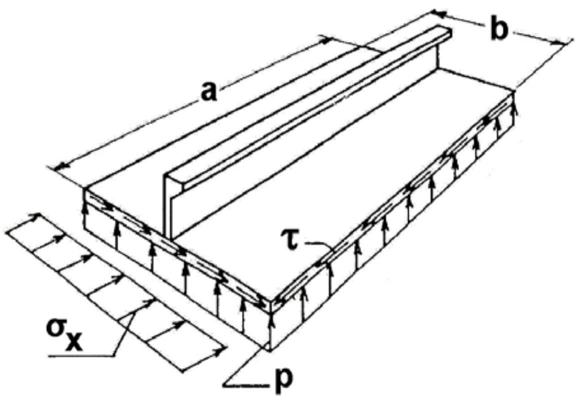


Fig. (6). Loading of a ship bottom structural element

Modes of failure of beam columns

The modes of failure of beam columns are shown in fig.(7).

The load deflection curves for a beam column under various end loading conditions is shown in Fig.(8)

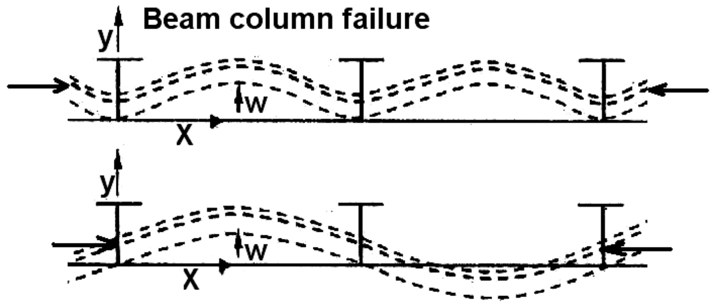


Fig.(7). Modes of failure of beam columns

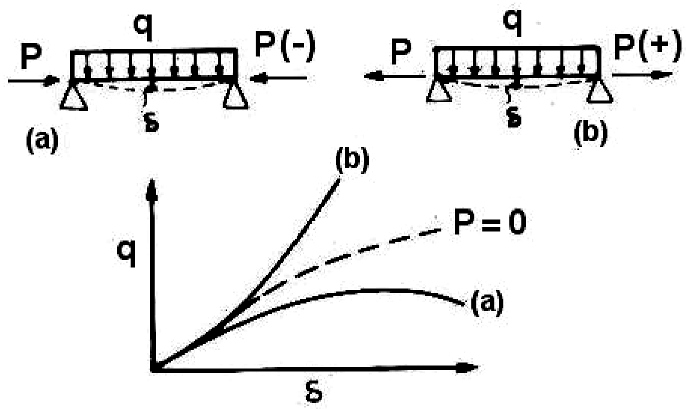


Fig.(8). Load-deflection curves of beam columns

Flexural buckling of beam columns

The section configuration used in the analysis of flexural buckling of longitudinals, stiffeners or frames should take the attached plating of the section into account. The geometrical characteristics of beam columns are shown in Fig.(9)

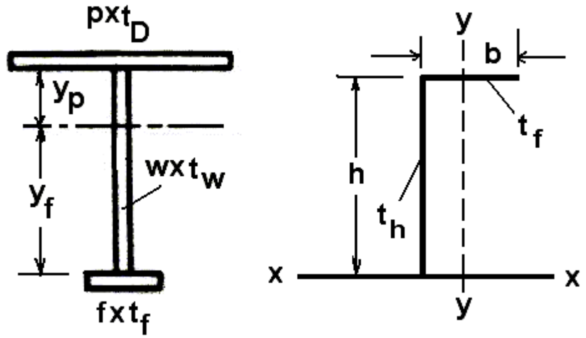


Fig.(9). Different section configurations of beam columns

The Euler critical buckling stress of a simply supported beam column of length L is given by:

$$\sigma_e = \frac{\pi^2 . EI}{AL^2}$$

Where: E = modulus of elasticity

I = least second moment of area

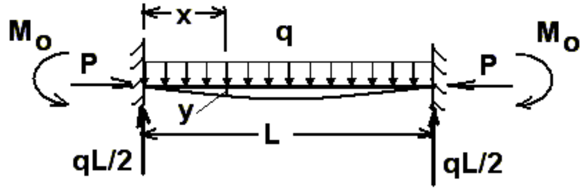
A = cross-sectional area of beam

A and I to take the effective breadth of plating " b_e", into account.

b_e = s, s =Longitudinal spacing.

Consider the following cases:

Case 1: Beam column under uniform loading and fixed at both Ends



$$M = P \cdot y + \frac{qL}{2} \cdot x - \frac{q \cdot x^2}{2} - M_o$$

Thus: $$EI \frac{d^2y}{dx^2} = -P \cdot y - \frac{qL}{2} \cdot x + \frac{q \cdot x^2}{2} + M_o$$

Let: $\mu = /$

Hence: $$\frac{d^2y}{dx^2} + \mu^2 y = \frac{q}{2EI}\left(xL - x^2\right) + \frac{M_o}{EI}$$

Then: $$y = A \cdot \sin \mu x + B \cdot \cos \mu x + \frac{M_o}{P} - \frac{q}{2P}\left(xL - x^2 + \frac{2}{\mu^2}\right)$$

At x = 0 y = 0

Then: $$B = \frac{q - \mu^2 M_o}{P \cdot \mu^2}$$

At x = L y = 0

Then: $$A = \frac{q - \mu^2 M_o}{P \cdot \mu^2}\left(\frac{1 - \cos \mu L}{\sin \mu L}\right)$$

Hence:

$$y = \frac{q-\mu^2 M_o}{P\cdot\mu^2}\left(\frac{1-\cos\mu L}{\sin\mu L}\right)\cdot \sin\mu x + \frac{q-\mu^2 M_o}{P\cdot\mu^2}\cdot\cos\mu x + \frac{\mu^2 M_o - q}{P\cdot\mu^2} - \frac{q}{2P}\left(Lx - x^2\right)$$

At x = 0 ′ = —— =

$$M_o = qL^2\left(\frac{2\tan\frac{\mu L}{2} - \mu L}{2\mu^2 L^2\cdot\tan\frac{\mu L}{2}}\right)$$

Thus: y_{max}. **(at x = L/2)**

$$= \frac{q-\mu M_o}{P\cdot\mu^2}\left[\cos\frac{\mu L}{2} + \frac{1-\cos\mu L}{2\cos\frac{\mu L}{2}} - 1\right] - \frac{qL^2}{8P}$$

$$= \frac{q-\mu M_o}{P\cdot\mu^2}\left[\frac{1}{\cos\frac{\mu L}{2}} - 1\right] - \frac{qL^2}{8P}$$

Substituting for $\cos\frac{\mu L}{2}$ **and** $\tan\frac{\mu L}{2}$ **, we get:**

$$\cos\frac{\mu L}{2} = 1 - \frac{\mu^2 L^2}{8} + ...$$

$$\tan\frac{\mu L}{2} = \frac{\mu L}{2} + \frac{\mu^3 L^3}{3} + ...$$

Hence: $\approx \frac{}{- /}$ **and** $M_o \cong \frac{qL^2/12}{1-P/P_E}$

Then: $M_{max} = P\cdot y_{max} + \frac{qL^2}{8} - M_o \approx \frac{1}{1-P/P_E}\left[P\cdot y_q - \frac{qL^2}{12}\right] + \frac{qL^2}{8}$

$$\sigma_{max} = \frac{P}{A} + \frac{1}{Z}\left[\frac{1}{1-P/P_E}\left(P\cdot y_q - \frac{qL^2}{12}\right) + \frac{qL^2}{8}\right]$$

<u>Summary of case 1:</u>

= · + —· − —— −

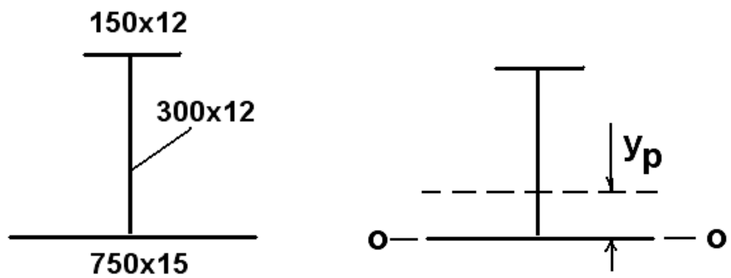
$= \frac{}{- /}$, = ———

$$M_o = \frac{qL^2/12}{1-P/P_E}$$

$$M_{max} = P\cdot y_{max} + \frac{qL^2}{8} - M_o$$

$$\sigma_{max} = \frac{P}{A} + \frac{M_{max}}{Z}$$

<u>Example</u>. Calculate the maximum compressive stress induced in a bottom longitudinal of an oil tanker, given that the ship section is subjected to a hogging moment of 300 MNm, E = 210 GN/m^2.



The section modulus of the tanker at the heel = 8m^3
The design draught = 10 m
Longitudinal spacing = 750 mm
Transverse spacing = 3 m
Longitudinal section, as shown, assume longitudinal hinged at both ends.
(1 ton = 9.964 kn)

Solution

$$\sigma_{max} = \frac{P}{A} + \frac{1}{Z}\left[\frac{1}{1-\frac{P}{P_e}}\left(P\cdot y_q - \frac{qL^2}{12}\right) + \frac{qL^2}{8}\right]$$

$$= \times \quad \times \gamma = \quad \times \quad \times \quad =$$

L =3 m

A = 0.75×0.015+0.15×0.012+0.3×0.012 = 0.01665 m2

$$\sigma = M/Z = \frac{300}{8} = 37.5 \quad MN/m^2$$

$$P = \sigma \cdot A = 37.5 \times 0.01665 = 0.624 \quad MN$$

$$P_e = \frac{4\pi^2 EI}{L^2}, \qquad I = I_o - A\cdot y_P^2$$

$$= \quad \times \quad \times \quad + \quad \times \quad \times\left(\frac{}{}\right)^2 + \frac{\times}{}$$

$$= \mathbf{0.00027}$$

$$= \frac{\times \quad \times \quad + \quad \times \quad \times}{} =$$

$$I = 0.00027 - 0.01665 \times 0.0648^2 = 0.0002 \quad m^4$$

$$P_e = \frac{4\pi^2 \times 210 \times 10^3 \times 0.0002}{3^2} = 184.0 \quad MN$$

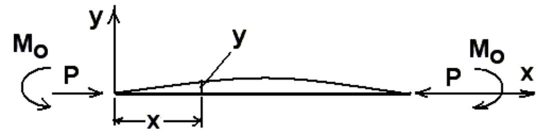
$$= \frac{}{} = \frac{\times \quad \times}{\times \quad \times \quad \times} =$$

$$\sigma_{max} = 37.5 + \frac{1}{8}\left[\frac{1}{1-\frac{0.624}{184}}\left(0.624\times 0.031 - \frac{7.6875\times 9.964\times 3^2}{12}\right) + \frac{7.6875\times 9.964\times 3^2}{8}\right]$$

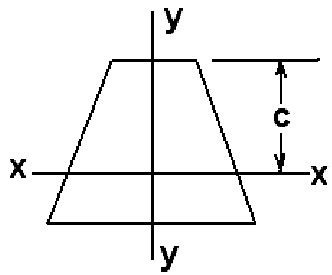
$$= \quad + -\left\{\frac{}{}\left(- \quad \right) + \quad \right\}$$

$$=$$

Case 2: Beam column subjected to end moments



$$= - \quad \cdot \quad -$$



Hence: $\frac{}{} = - \quad \cdot \quad -$

Let: $\frac{P}{EI} = \mu^2$

Then: $\frac{d^2y}{dx^2} + \mu^2 y = -\frac{M_o}{EI}$

Solution: $\mathbf{y = A\cdot \sin \mu x + B\cdot \cos \mu x - \frac{M_o}{P}}$

At x = 0 y = 0 , Then: $= \frac{}{}$

At x = L y = 0 Then: $A = \frac{M_o}{P}\left(1 - \cos \mu L\right)/\sin \mu L$

Hence, the deflection equation is given by:

$$y = \frac{M_o}{P}\left(\frac{1-\cos\mu L}{\sin\mu L}\right)\sin\mu x + \frac{M_o}{P}\left(\cos x - 1\right)$$

M_{max} occurs at $= \frac{}{}$

$$M_{max} = P \cdot y_{max} + M_o = \frac{M_o}{1-\frac{P}{P_E}},$$

Where: $P_E = \pi^2 E I / L^2$

$$\sigma_{max} = \frac{P}{A} + M_{max} \cdot \frac{c}{I} = \sigma + \frac{M_o}{1-\frac{P}{P_E}} \cdot \frac{c}{I}$$

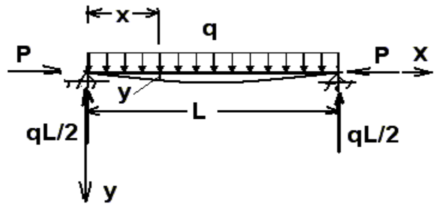
Let: $\sigma_o = \frac{M_o \cdot c}{I}$

Then: $$\sigma\left[1 + \frac{\sigma_o}{\sigma\left(1-\frac{P}{P_E}\right)}\right] \le \sigma_y$$

Case 3: Beam column hinged at both ends and subjected to uniform loading

The bending moment at a distance 'x" from the left support is given by:

$= \cdot + \frac{}{} \cdot - \frac{}{}$



$$y_{max} = \frac{y_q}{1 - P/P_E}$$

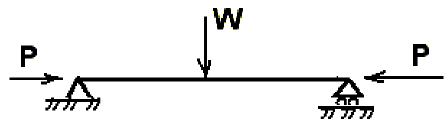
$= \frac{}{}$

$= \cdot + \frac{}{}$

$$\sigma_{max} = \frac{P}{A} + \frac{M_{max}}{Z}, \quad = /$$

Summary of beam column formulae

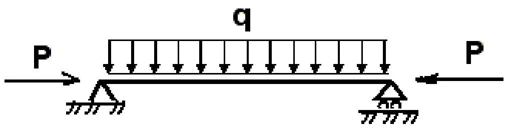
Case 1: Beam hinged at both ends and subjected to a concentrated load



$$y_{max} = \frac{WL^3}{48EI}\left[\frac{3(\tan\mu - \mu)}{\mu^3}\right] = \frac{WL^3}{48EI} \cdot \varphi_1$$

$$M_{max} = \frac{WL}{4}\left[\frac{3(\tan\mu)}{\mu}\right] = \frac{WL}{4} \cdot \varphi_2$$

<u>Case 2: Beam hinged at both ends and subjected to uniform loading</u>



$$y_{max} = \frac{5qL^4}{384EI}\left[\frac{12}{5}\frac{\left(2\sec\mu - 2 - \mu^2\right)}{\mu^4}\right] = \frac{5qL^4}{384EI}\varphi_3$$

$$M_{max} = \frac{qL^2}{8}\left[\frac{2\left(\sec\mu - 1\right)}{\mu^2}\right] = \frac{qL^2}{8}\varphi_4$$

<u>Case 3: Beam hinged at both ends and subjected to end moments</u>

$$y_{max} = \frac{M_oL^2}{8EI}\left[\frac{2\left(\sec\mu - 1\right)}{\mu^2}\right] = \frac{M_oL^2}{8EI}\varphi_4$$

$$M_{max} = M_o\left[\sec\mu\right] = M_o\varphi_5$$

The different "φ" values are given in the following table:

/	/(− /)					
0	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
0.1	**1.111**	**1.109**	**1.091**	**1.107**	**1.114**	**1.137**
0.2	**1.250**	**1.247**	**1.205**	**1.253**	**1.258**	**1.310**
0.3	**1.429**	**1.420**	**1.350**	**1.430**	**1.441**	**1.533**
0.4	**1.667**	**1.657**	**1.545**	**1.663**	**1.684**	**1.831**
0.5	**2.000**	**1.982**	**1.815**	**2.000**	**2.029**	**2.252**
0.6	**2.500**	**2.477**	**2.223**	**2.502**	**2.544**	**2.884**
0.7	**3.333**	**3.303**	**2.901**	**3.347**	**3.407**	**3.942**
0.8	**5.000**	**4.943**	**4.253**	**5.013**	**5.124**	**6.057**
0.9	**10.000**	**9.876**	**8.308**	**10.040**	**10.290**	**12.420**

Buckling of ship plating

Plate configurations

The different plate configurations commonly used in ship structure are shown in fig.(10)

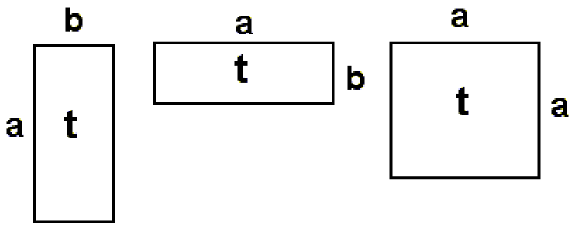


Fig.(10). Common ship plate configurations

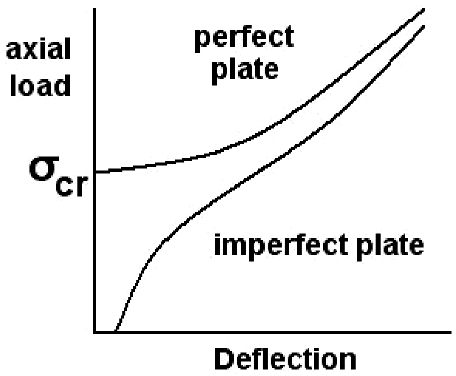


Fig. (11). Load deflection curves for a perfect/imperfect plate

Buckling of ship plating

A long plate of length "a" and width "b" subjected to in-plane compressive stresses buckles into a number of square panels if the length "a" is an exact number of the width "b". The axial

load deflection curves for a perfect plate and an imperfect plate is shown in Fig.(11).

The common causes of plate buckling of ship structures are:

- **High Compressive Stresses**
- **High Shear Stresses**
- **Presence of High Compressive Residual Stresses**
- **Presence of Combined Stresses**
- **Lack of Adequate Flexural Rigidity**
- **Lack of Adequate Stiffening**
- **Extensive and /or Improper Use of HTS**
- **Excessive Material Wastage Due to General and for Local Corrosion**

Plate deformation under compressive loadings

The shape of plate deformation depends on:

- **Type of loading**
- **Type of end supports**
- **Aspect ratio of plate**

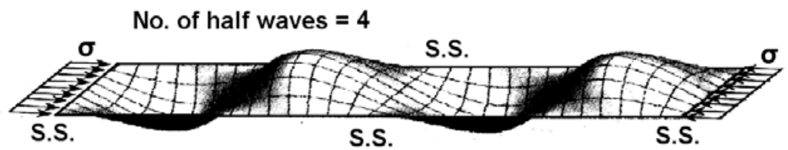


Fig.(12).Plate deformation under end compressive stresses, all edges simply supported

Typical plate deformations under end compressive stresses, shear stresses and different end support conditions are shown in Figs.(12,13,14,15,16,17)

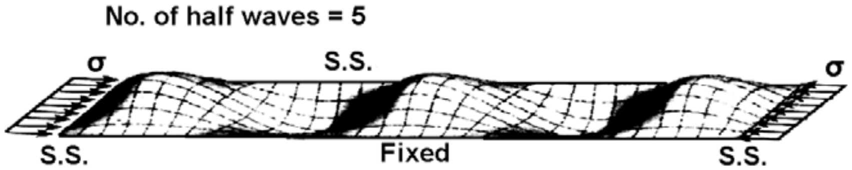


Fig.(13).Plate deformation under end compressive stresses, one edge fixed and the other free

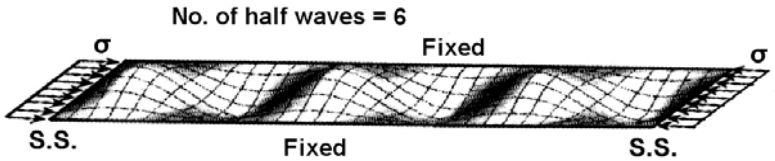


Fig.(14).Plate deformation under end compressive stresses, both edges are fixed

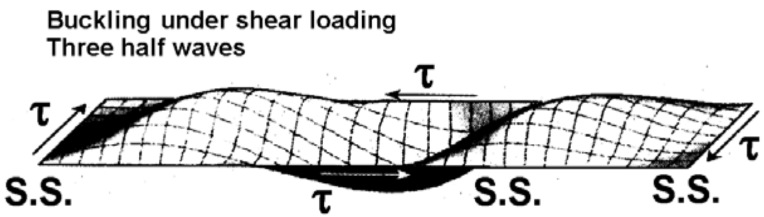


Fig.(15).Plate deformation under shear stresses, all edges simply supported

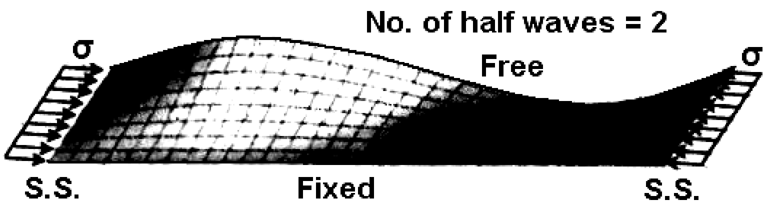


Fig.(16).Plate deformation under end compressive stresses and one edge free and one edge fixed

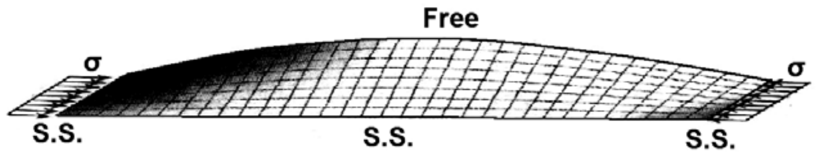


Fig.(17).Plate deformation under end compressive stresses and one edge free and one edge simply supported

Plate deformation under in-plane compressive stresses could be idealized as shown in Fig.(18)

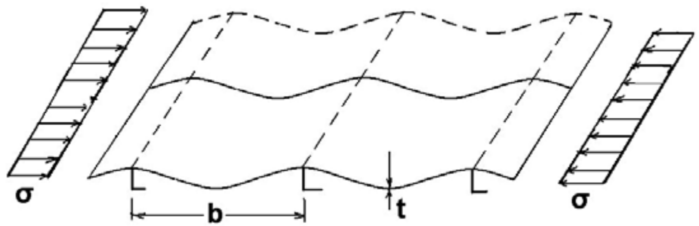


Fig.(18). Typical plate deformation under in-plane compressive stresses

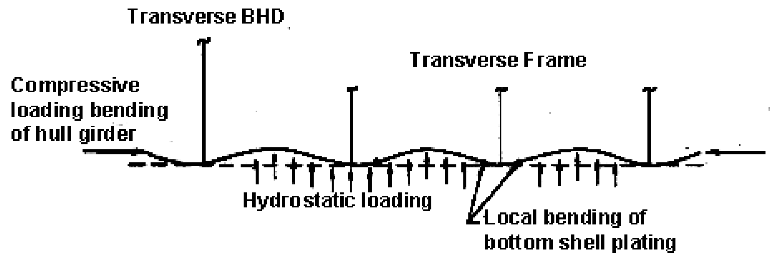


Fig.(19). Typical loadings and deformations of ship bottom plating

Plate deformation under in-plane stresses and later loading is shown in Fig.(19)

Plate deformation under in-plane loading and assuming fixed ends is shown in Fig.(20).

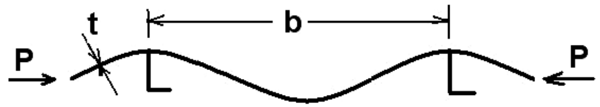


Fig.(20). Plate assumed fixed at both ends

Plate deformation under in-plane loading and assuming simply supported ends is shown in Fig.(21).

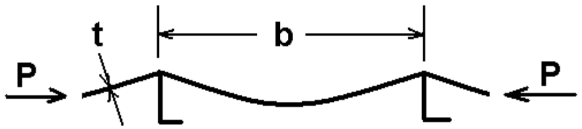


Fig.(21). Plate assumed simply supported at both ends

Critical Buckling Stress of plating

The Euler elastic buckling equation is given by:

$$= \frac{}{(- \upsilon)}\left(- \right) .k$$

The magnitude of the factor "k" depends on:

- **Aspect ratio and geometry of plate**
- **Boundary support conditions, see Fig.(22).**
- **Type of loading**

Assessment of buckling strength of plating under in plane loading and different boundary support conditions

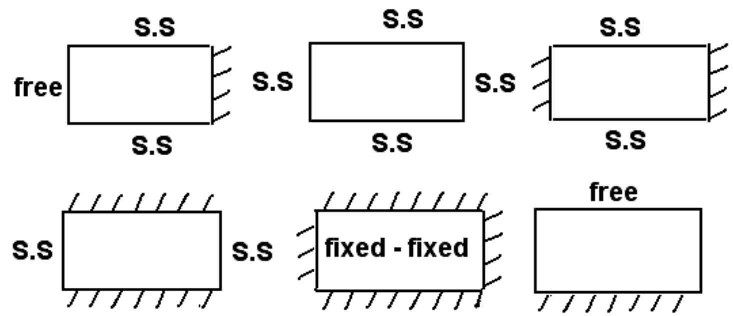


Fig.(22). Different plate boundary support conditions

The plate boundary conditions commonly assumed in the analysis of buckling strength of plating of ship structure are: fixed, simple support or free edge, see Fig. (22).

Case 1: Plate fixed at both long edges and subjected to in-plane compressive stresses

The Euler buckling equation is given by:

$$= \frac{}{(- \upsilon)}\left(- \right)\left[\left(- \right) + \right]$$

b = length of shorter edge of plate

For a = b and substituting for ν = 0.3, we get:

$$= \frac{\quad}{(\ -\upsilon\ )}\left(\frac{}{}\right) \cong 3.6E\left(\frac{}{}\right)$$

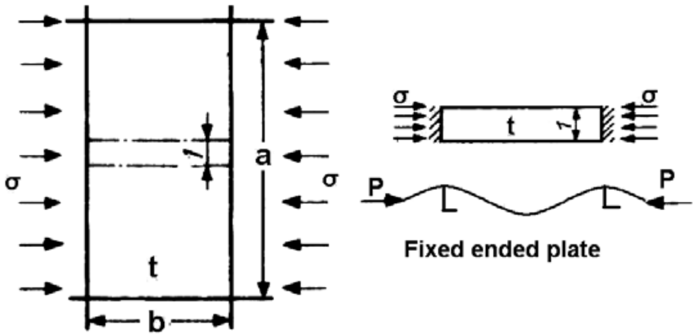


Fig.(23) Simply supported and fixed ended plate strip

Consider a strip of width 1.0 m, see Fig.(23). The critical buckling stress of the strip is given by:

$$= \frac{\quad}{\quad} = \frac{\quad}{}\left(\frac{}{}\right)$$

<u>Case 2: Plate simply supported at both long edges and subjected to in-plane compressive stresses</u>

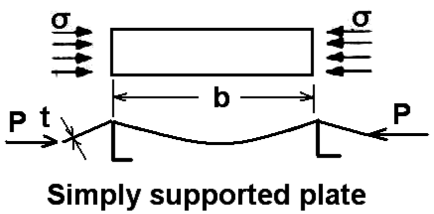


Fig. (24). Deformed shape of a simply supported plate

Consider a strip of width 1.0 m, see Fig.(24). The critical buckling stress of the strip is given by:

$$= \frac{\quad}{-\upsilon}\left(\frac{}{}\right) = 0.9\ E\left(\frac{}{}\right)^2 \quad \text{for } a >> b$$

<u>Case 3: Plate has one edge free</u>

The Euler buckling equation for a plate having one edge free and subjected to end compressive stresses is given by:

$$= \frac{\quad}{(\ -\upsilon\ )}\left(\frac{}{}\right)$$

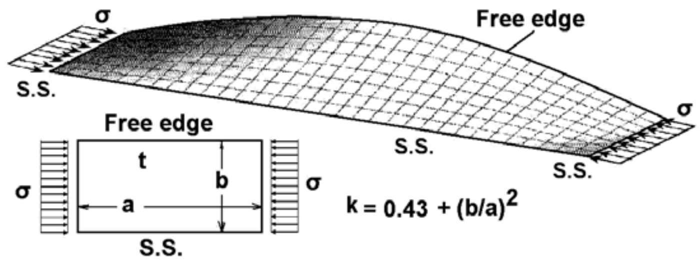


Fig.(25)Buckling of a plate with one edge free and the other simply supported

The constant k depends on the boundary support condition of the other edge of the plate as follows:

i- <u>Simply supported edge, see Fig.(25)</u>

See Fig. (24), k = 0.43 + (b/a)^2

ii- Fixed edge

See Fig.(26), k = 1.28

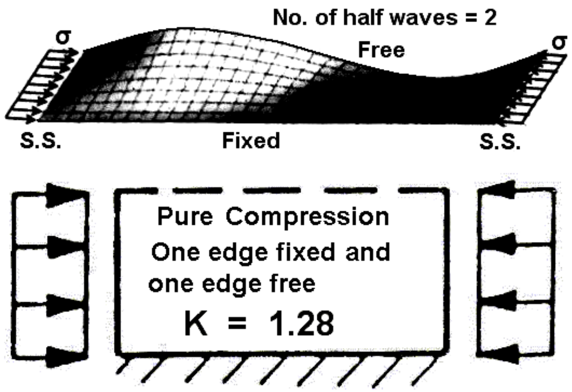


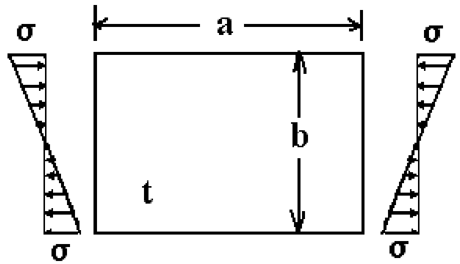
Fig.(26)Buckling of a plate with one edge free and the other fixed

Buckling of simply supported plating under various in plane loading conditions

Case 1: Plate subjected to pure bending stresses

The Euler buckling equation for a plate subjected to in-plane bending stresses is given by:

$$\sigma_{eb} = \frac{\pi^2 E}{12(1-\upsilon^2)}\left(\frac{t}{b}\right)^2 k_b$$



σ_{eb} = Euler buckling stress under pure bending

The factor k_b depends on the a/b ratio. For typical panels of ship structure, the constant k_b= 24.0

Case 2: Plate subjected to pure shear stresses

Buckling due to in-plane shear loading causes wrinkling of the plate at 45 degrees, see Fig.(27).

The critical buckling stress given by the Euler buckling equation for a plate subjected to in-plane shear stress is given by:

$$\tau_{es} = \frac{\pi^2 E}{12(1-\upsilon^2)}\left(\frac{t}{b}\right)^2 k_s$$

τ_{es}= Euler buckling stress under pure shear

The constant k_s is given by:

$k_s = 4.8 + 3.6(b/a)^2$ for simply supported edges

$k_s = 8.1 + 5.1(b/a)^2$ for clamped edges

For typical ship structure plate panels, we have:

b/a = 1/3-1/5

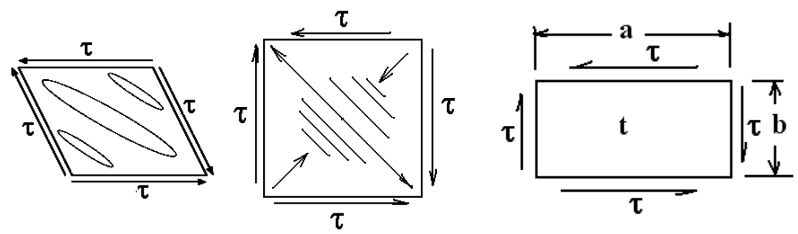


Fig.(27) Buckling of a simply supported plate in shear

Hence $(b/a)^2 = 0.04 - 0.1$ with a mean value = 0.07
Then for a simply supported plate, k = 5.05
And for a fixed-fixed plate. K = 8.45
The critical shear buckling stress is given by:

$\tau_{cr} = \tau_{es}$ **when** $\tau_{es} < 0.5\tau_y$ **, where:** $\tau_y = \dfrac{\sigma_y}{\sqrt{3}}$

$$\tau_{cr} = \frac{\sigma_y}{\sqrt{3}}\left[1 - \frac{\sigma_y}{4\sqrt{3}.\tau_{es}}\right], \text{ when } \tau_{es} > 0.5\tau_y$$

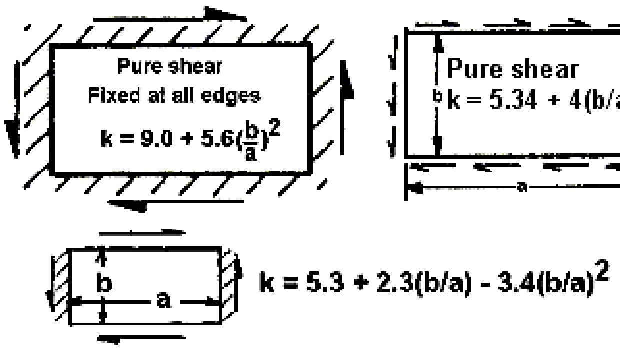


Fig.(28). K factors for different boundary conditions

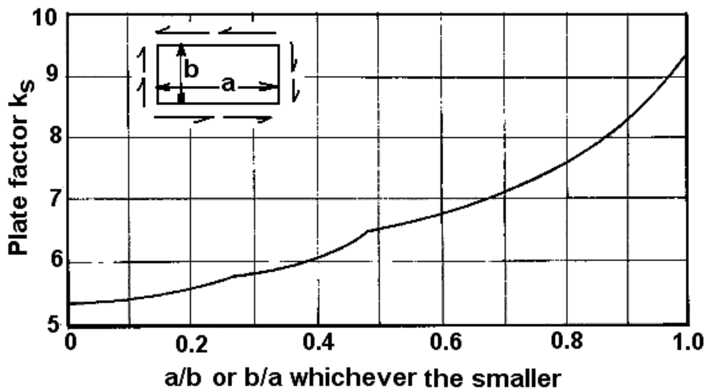
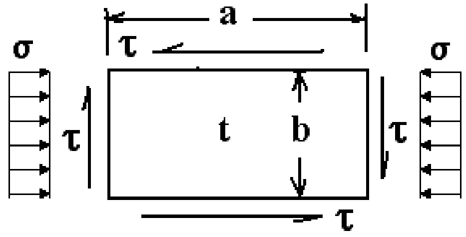


Fig.(29).Buckling factor for a plate under shear loading

<u>Assessment of buckling strength of plating subjected to combined loading</u>

The following interaction formulae are used to evaluate the critical buckling stresses for plates subjected to a combination of in plane compressive, bending and shear stresses.

i- <u>Combined shear and pure compression</u>



The plate buckling equation is given by:

$$\left(\frac{\tau}{\tau_{es}}\right)^2 + \frac{\sigma}{\sigma_{ec}} \leq 1.0$$

τ = applied stress

τ_{es} = Euler critical shear buckling stress

$$\tau_{cr} = \frac{\pi^2 E}{12(1-\upsilon^2)}\left(\frac{t}{b}\right)^2 \cdot k_s \quad \text{for } \tau_{es} < 0.5\tau_y$$

$$= \tau_y\left(1 - \frac{\tau_y}{4\tau_{es}}\right) \quad \text{for } \tau_{es} > 0.5\tau_y$$

σ = applied stress

$$\sigma_{ec} = \frac{\pi^2 E}{12(1-\upsilon^2)}\left(\frac{t}{b}\right)^2 \cdot k_c \quad \text{for } \sigma_{ec} \leq 0.5\,\sigma_y$$

$$= \sigma_y \left(1 - \frac{\sigma_y}{4\sigma_{ec}}\right) \qquad \textbf{for } \sigma_{ec} \geq 0.5\,\sigma_y$$

k_s = buckling factor depending on the boundary conditions
The buckling curve for the combined shear and in plane compression is shown in Fig.(30)

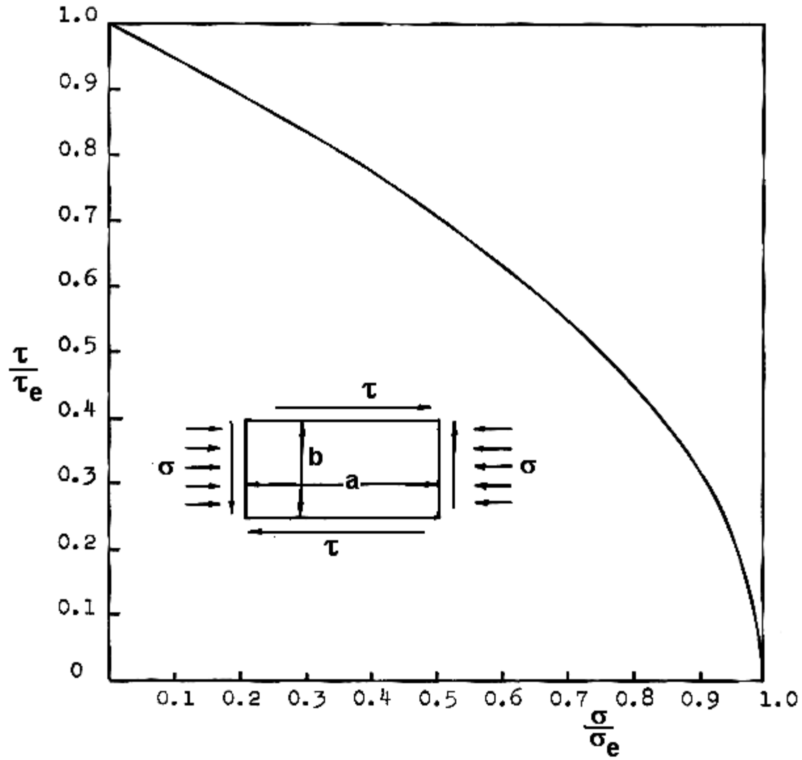


Fig.(30).Condition of combined shear and in plane loading

ii- C<u>ombined shear and in-plane bending</u>

Plate buckling equation is given by, see Fig.(31):

$$\left(\frac{\tau}{\tau_{es}}\right)^2 + \left(\frac{\sigma_b}{\sigma_{eb}}\right)^2 \leq 1.0$$

Where: σ = applied bending stress

$$\sigma_{eb} = \frac{\pi^2 E}{12(1-\upsilon^2)}\left(\frac{t}{b}\right)^2 \cdot k_b \qquad \textbf{for } \sigma_{eb} \leq 0.5\,\sigma_y$$

$$= \sigma_y \left(1 - \frac{\sigma_y}{4\sigma_{eb}}\right) \qquad \textbf{for } \sigma_{eb} \geq 0.5\,\sigma_y$$

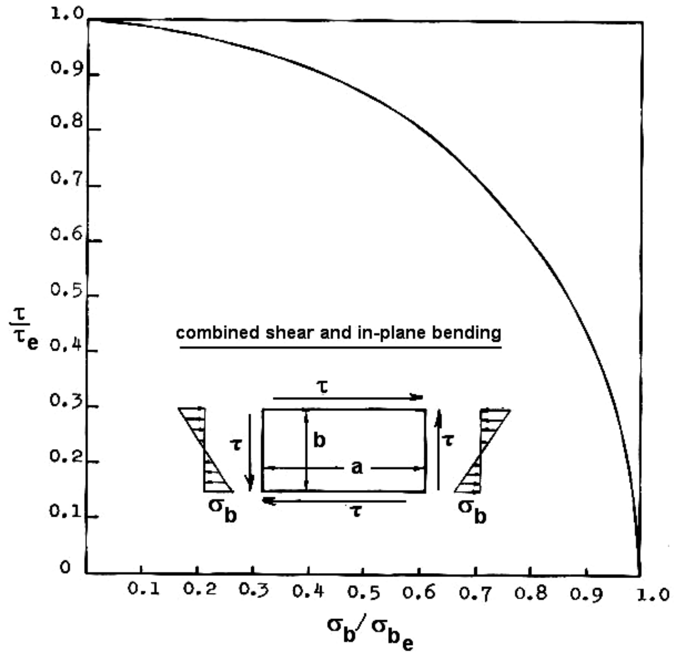


Fig.(31). Condition of combined shear and in plane bending

iii- <u>Non-uniform compression (Combined in-plane bending and compression)</u>

Plate buckling equation is given by see Fig.(32):

$$\frac{\sigma}{\sigma_{ec}} + \left(\frac{\sigma_b}{\sigma_{eb}}\right)^2 \leq 1.0$$

Where: $\sigma = \frac{\sigma_1 + \sigma_2}{2}$, $\sigma_b = \frac{\sigma_1 - \sigma_2}{2}$

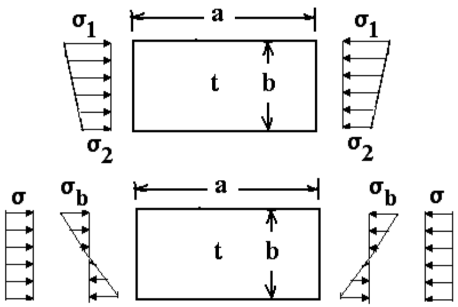


Fig.(32).Plate subjected to in-plane compression and bending

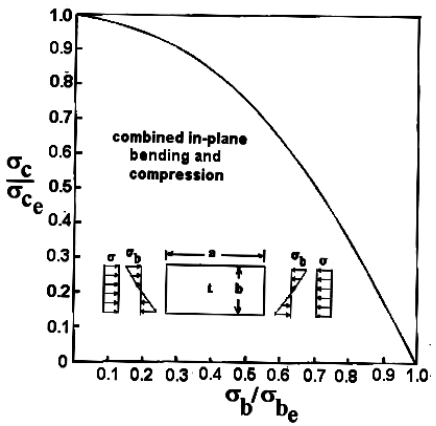
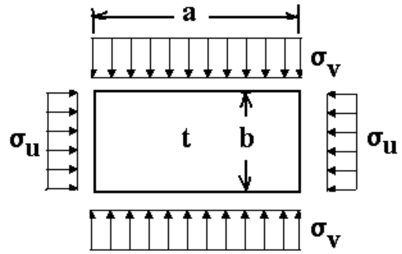


Fig.(33) Plane bending and compression

iv- <u>In plane compression in two orthogonal directions</u>

Plate buckling equation is given by:

$$\frac{\sigma_\mu}{\sigma_{e\mu}} + \frac{\sigma_\upsilon}{\sigma_{e\upsilon}} \leq 1.0$$



<u>v- Combined shear, in-plane bending and compression</u>

Plate buckling equation is given by, see Fig.(33):

$$\frac{\sigma}{\sigma_{ec}} + \left(\frac{\sigma_b}{\sigma_{eb}}\right)^2 + \left(\frac{\tau}{\tau_{es}}\right)^2 \leq 1.0$$

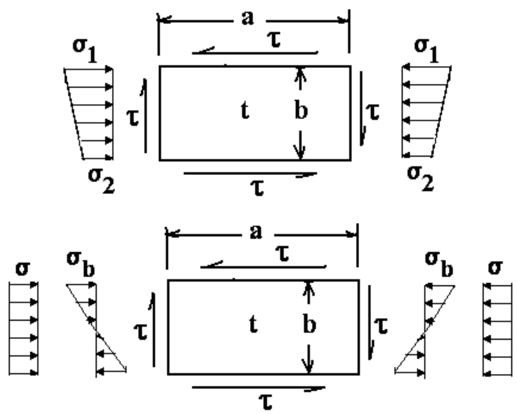
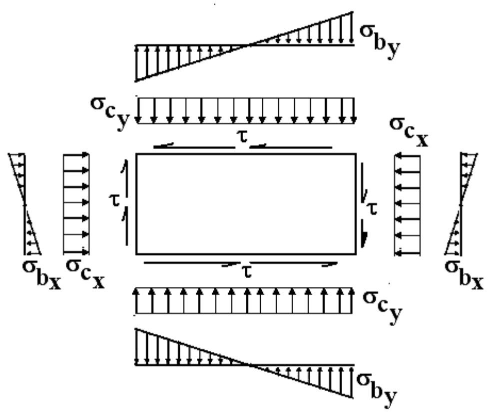


Fig.(34).Plate subjected to in-plane compression, bending and shear stresses

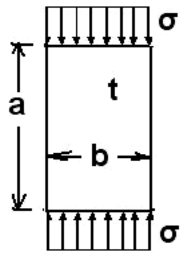


Fig(35).combined shear, in plane and bending over all edges

Effect of the magnitude of the applied stresses

Let: σ_{cr} = critical buckling stress

Then: when: $\sigma_e \leq \sigma_y/2$



$$\sigma_{cr} = \sigma_e = \frac{k\pi^2 E}{12(1-\upsilon^2)}\left(\frac{t}{b}\right)^2$$

Where: σ_e = Euler buckling stress

σ_y = yield stress

When: $\sigma_e > \sigma_y/2$ $\quad \sigma_{cr} = \sigma_y\left[1 - \frac{\sigma_y}{4\sigma_e}\right]$

This is Johnson-Ostenfeld formula.

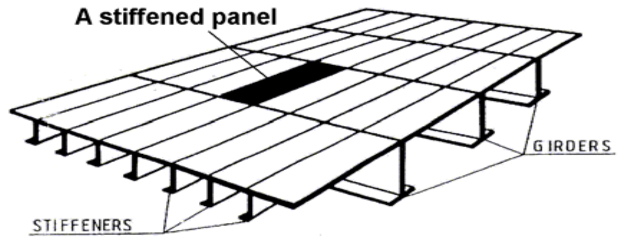
Also: when: $\tau_e \leq \_\, \tau_y$, $\tau_y = \frac{\sigma_y}{\sqrt{3}}$

When: $\tau_e \geq \_\, \tau_y$, $\tau \quad = \tau\left(\quad - \frac{\tau}{\tau}\right)$

Where: τ_e = Euler buckling stress under shear loading

Buckling of stiffened panels

The basic structural configuration of a stiffened panel is shown in Fig.(36)



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ig.(36). A typical stiffened panel

In-plane loading conditions of stiffened panels

The most common in-plane loading conditions of stiffened panels of ship structure are shown in Fig.(36)

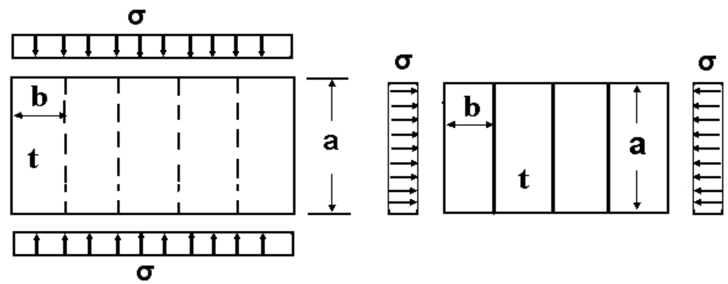


Fig.(37). The two main loading conditions of stiffened panels

Modes of failure of stiffened panels

The general modes of failure of stiffened panels are:

- Buckling of plate panel between stiffeners, see Fig.(38)

- Flexural buckling of stiffeners

- Torsional buckling of stiffeners

- Lateral buckling of stiffeners

- Flexural buckling of plate-stiffener combination (Global buckling of stiffened panel).

The different modes of deformation of a transversely framed panel of plating subjected to end compressive stresses are shown in Fig. (38). The torsional rigidity of the stiffening members affects significantly the shape of the panel deformation. The modes of deformation under lateral loading for symmetrical and asymmetrical sections are shown in Fig.(39).

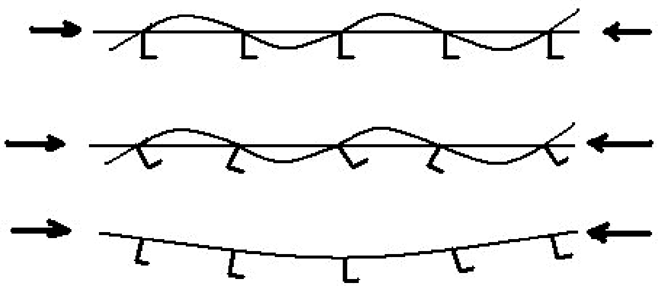


Fig.(38). Different modes of failure of a transversely stiffened panel of plating

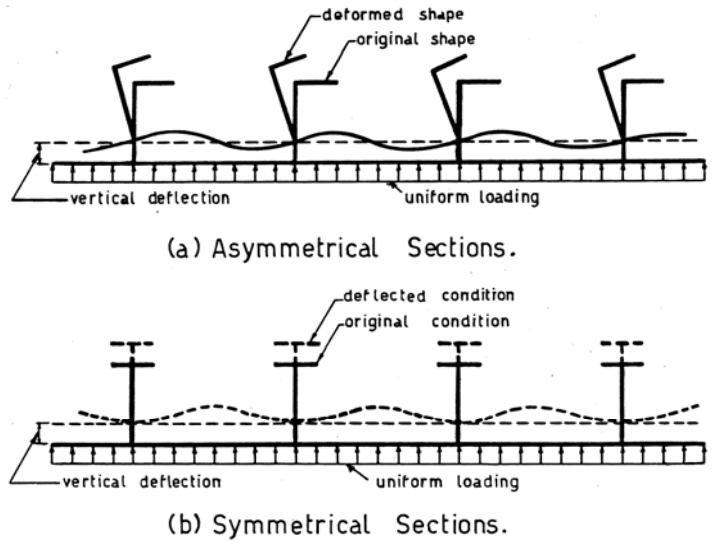


Fig.(39). Flexural bending and torsional buckling under lateral loading

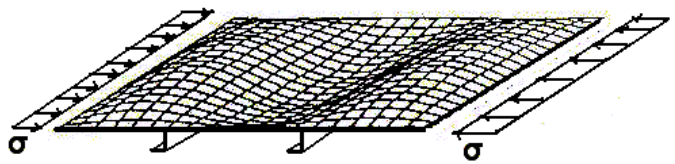


Fig.(40). Plate buckling of a transversely framed panel

1- Flexural buckling of plate panel

Plate buckling of a transversely framed panel is shown in Fig.(40). The deformation shape of the buckled plate depends on the type of loading and the assumed end support conditions.

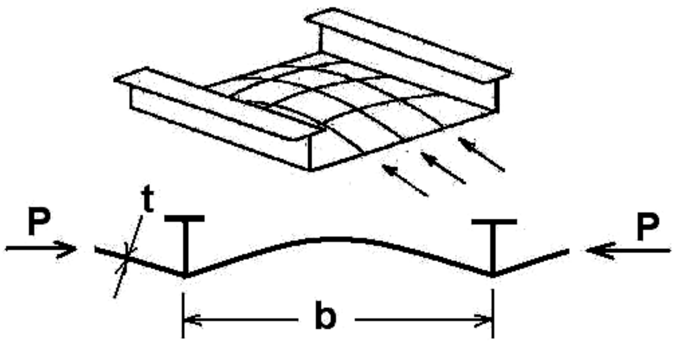


Fig.(41). Deformation shapes for simple end supports

Fig.(41). Shows the buckled shape of a plate subjected to in-plane stresses and for simple end supports. Fig. (42). Shows the buckled shape of a plate subjected to in-plane stresses and for fixed end supports.

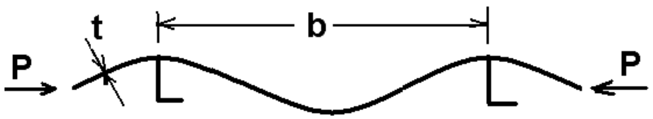


Fig.(42). Deformation shape for fixed end supports

a- <u>Critical buckling stress of plating of stiffened panels subjected to in-plane end compressive stresses</u>

<u>1- Longitudinally stiffened panels</u>

Let: a = longitudinal spacing

b = length of panel, see Fig.(43)

The Euler buckling stress for a panel subjected to end in-plane compressive stresses and fixed at both ends is given by, see Fig.(43):

$$= \frac{\quad}{\quad},$$

$$= \frac{\quad}{\left(\quad - \upsilon\quad\right)}$$

$$\sigma_e \cong 3.6E\left(\frac{\quad}{\quad}\right)$$

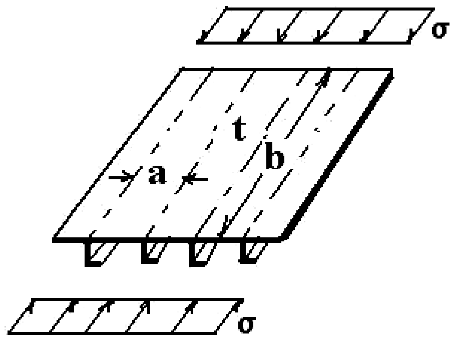


Fig.(43) Buckling of plating of a longitudinally stiffened panel

The Euler buckling stress for a panel simply supported at both ends is given by:

$$= \frac{\quad}{\quad}, \quad D = \frac{Et^3}{12\left(1-\upsilon^2\right)}$$

$$\sigma_e \cong 0.9E\left(\frac{\quad}{\quad}\right)$$

The critical buckling stress σ_{cr} is given by:

$$\sigma_{cr} = \sigma_e \quad \textbf{for}\ \sigma_e \le 0.5\,\sigma_y$$

$$= \sigma_y\left(1 - \frac{\sigma_y}{4\sigma_e}\right) \quad \textbf{for}\ \sigma_e \ge 0.5\,\sigma_y$$

Where: σ_y = yield stress of the material

2- Transversely stiffened panels

Let: b = beam spacing

a = length of plate, See Fig.(44).

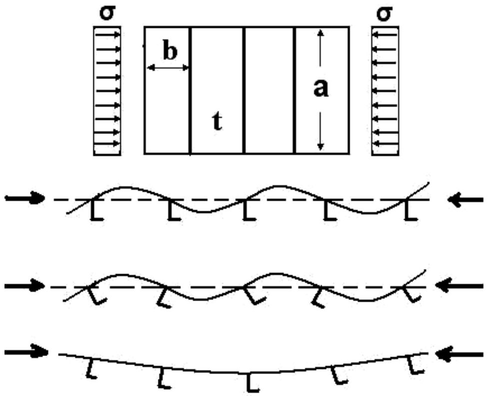


Fig.(44) Buckling of plating of a transversely stiffened panel

The Euler buckling stress for a simply supported panel subjected to in plane compressive stresses along the long edges is given by, see Fig.(44):

$$= \frac{}{}\left(+ \frac{}{}\right) , \quad = \frac{}{(\; - \upsilon \;)}$$

For ship structure, $— \approx \frac{1}{3} - —$, then $\sigma_e \approx \frac{\pi^2 D}{b^2 t}{}^2$

The critical buckling stress for a plate of the stiffened panel is given by: $\sigma_{cr} = \sigma_e$ for $\sigma_e \leq \sigma_y$

$$= \sigma_y \left(1 - \frac{\sigma_y}{4\sigma_e}\right) \quad \text{for } \sigma_e \geq 0.5\,\sigma_y$$

Note: $(\;)$longitudinal stiffening $\cong 4(\sigma_e)$ transverse stiffening

2- Buckling of stiffeners

The common buckling modes of stiffeners are:

- Flexural buckling
- Torsional buckling of the stiffeners
- Torsional buckling of the flange of the stiffener
- Lateral buckling of the web of the stiffener
- Lateral buckling of the flange of the stiffener

i- Flexural buckling of stiffeners

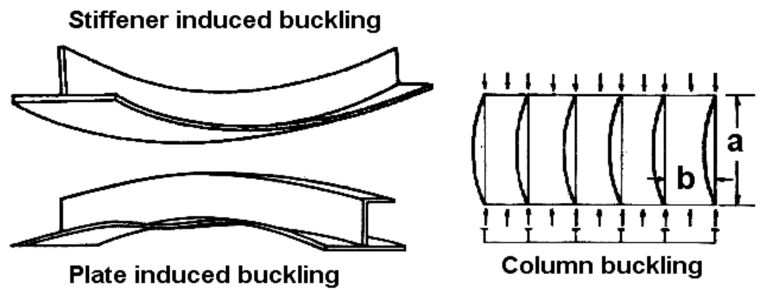


Fig.(45). Flexural buckling of stiffeners (beam-column mode)

The flexural critical buckling stress of stiffeners subjected to end compressive loads are obtained using the following generalized Euler formula, see Fig.(45):

$$\sigma_E = \frac{\pi^2 EI}{A\ell^2}$$

σ_E = Euler buckling stress and is given by:

For $\sigma_E \leq 0.5\,\sigma_y$, $\sigma_{cr} = \sigma_E$

For $\sigma_E \geq 0.5\,\sigma_y$ $\quad \sigma_{cr} = \sigma_y \left[1 - \dfrac{\sigma_y}{4\sigma_E}\right]$

Where: σ_y = lower yield stress

σ_{cr} = critical buckling stress

σ_E = Euler buckling stress

P_E = Euler buckling load

A = sectional area of column

$\ell =$ kl = Effective length of column

k = a factor depending on the type of end supports

k = 1.0 for hinged supports

k = 0.5 for fixed end supports

ii- Torsional buckling of stiffeners

The Euler stress for torsional buckling of a beam of length L and section configurations as shown in Fig.(46) is given by:

$$\sigma_e = \beta \frac{C \cdot \frac{\pi^2}{L^2} \cdot E\Gamma + GJ}{I_\rho} \qquad \text{Where:} \quad \Gamma = I_y \left(h + \frac{t_f}{2}\right)^2$$

G = shear modulus = E/2.6

I_P= polar m.o.i $= I_X + I_y$

= m.o.i of cross –section about y-y axis without effective plate flange

I_X = m.o.i of cross-section about x-x axis without effective plate flange

J = torsion constant of section

$$= -(\ \cdot\ +\ \times\ )$$

C = constant depending on the support condition at the ends of the stiffener.

C = 1.0 for simply supported ends

= 4.0 for fixed ends

β= a constant depending on the stiffener support conditions along the plate

β = 1.0

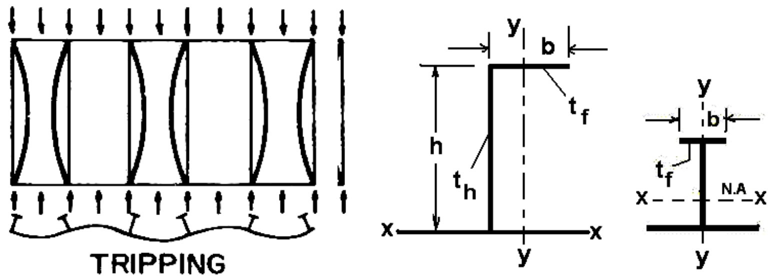


Fig.(46). Torsional buckling mode of failure for a longitudinally framed panel

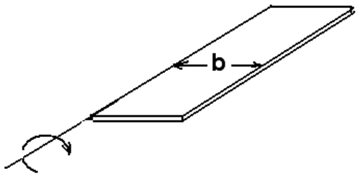
Torsion buckling of the flange

The torsion buckling stress of the top flange of a section is given by: $\sigma_e = \frac{GJ}{I_y}$

Where: I_y = m.o.i of Flange about y-y axis

J = torsion constant of section,

$J = b\,t_f^3/3$



For offset flange: $= \cdot$ —

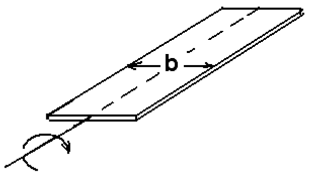
For T-stiffener: $= \cdot /$

Hence, the torsion buckling stress for an offset flange is given by:

$$\sigma_e = G \cdot \frac{b\,t_f^3/3}{t_f \cdot b^3/3} = G \cdot \left(\frac{t_f}{b}\right)^2$$

The torsion buckling stress for the flange of a T-stiffener is given by:

$$\sigma_e = G \cdot \frac{bt_f^3/3}{t_f \cdot b^3/12} = 4G \cdot \left(\frac{t_f}{b}\right)^2$$



iii- Lateral buckling of stiffeners

1- Lateral buckling of the web plate of a section

The Euler lateral buckling stress for the web plate of a section, assuming the web plate simply supported is given by, see Fig. (47):

$$\sigma_e = \frac{k\pi^2 E}{12(1-\upsilon^2)} \cdot \left(\frac{t}{b}\right)^2$$

Where: k= 4.0 when L/b >> 1.0

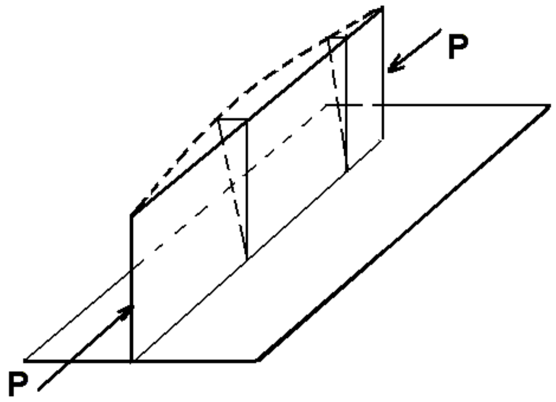


Fig.(47).Lateral buckling of the unflanged web beam column

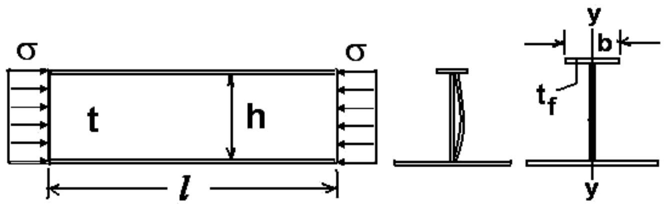


Fig.(48).Lateral buckling of the web of a flanged beam column

2- Lateral buckling of the flange of a beam column

The lateral buckling stress of the flange of a beam column is given by, see Figs.(48):

$$\sigma_e = \frac{\pi^2 E I_y}{L^2 A}$$

Where: I_y = m.o.i of flange about y-y axis.

A = cross–section area of flange

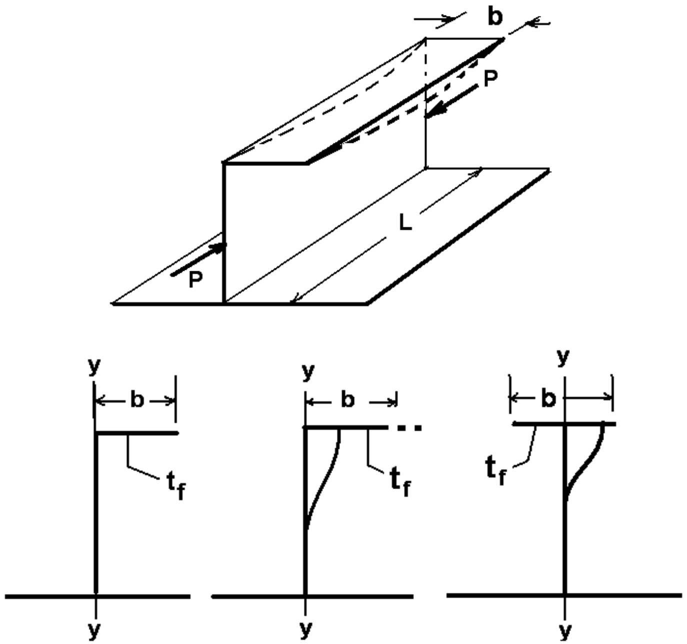


Fig.(49). Lateral buckling of the flange of a beam column

Case 1: T-Stiffener $\sigma_e = \frac{\pi^2 E}{12} \cdot (\frac{b}{L})^2$

Case2: Offset Flange $= \frac{\pi^2 E}{3} \cdot (\frac{b^2}{L^2})$

Gross panel buckling

The buckling mode of a transversely framed panel could be either a single mode of flexural plate buckling or a multiple mode of flexural plate buckling and torsional stiffener buckling, see Fig.(50) or a flexural, torsional and lateral buckling, see Fig.(51). An example of gross panel buckling is shown in Fig.(52) for the deck structure of a general cargo ship.

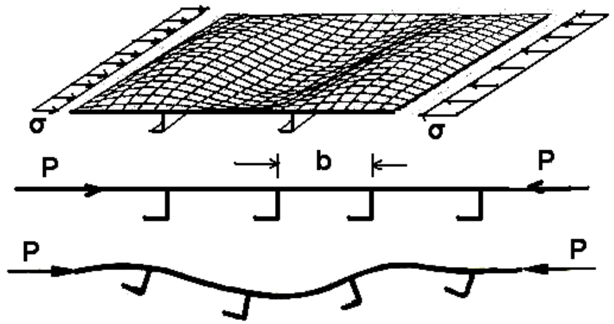


Fig.(50). Flexural and torsional buckling

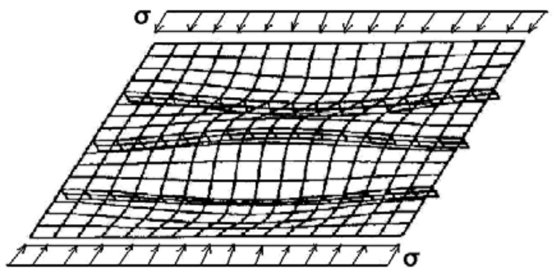


Fig. (51) Flexural, lateral and torsional buckling (tripping) of a stiffened panel.

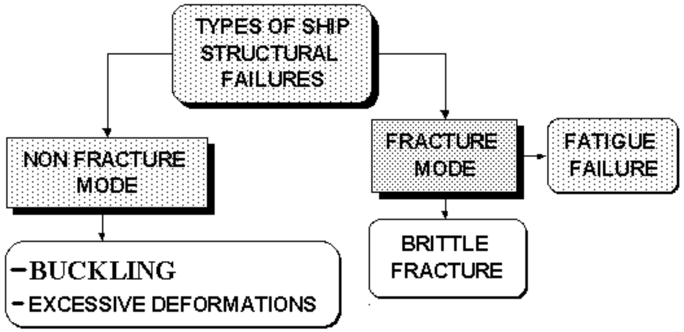
Fig.(52). Gross panel buckling of the deck structure of a general cargo ship

Types and causes of Ship structural failures

Types of ship structural failures

Ship structural failures are nearly always nonlinear, either a geometric nonlinearity (buckling or large deflections) or a material nonlinearity (yielding and plastic deformation). For steel members, the three basic types of failure and their subdivisions are as follows, see Fig.(53):

- o Large local plasticity
- o Instability Fracture,
- o Direct (tensile rupture), fatigue, brittle.



``Fg.(53) Types of ship structural failure

Causes of ship structural failures

The main causes of ship structural failures are, see Fig. (54):

- Design errors
- Material errors
- Fabrication residual stresses, distortions and errors, see Fig. (55, 56, 57).
- Operational, maintenance and repair errors

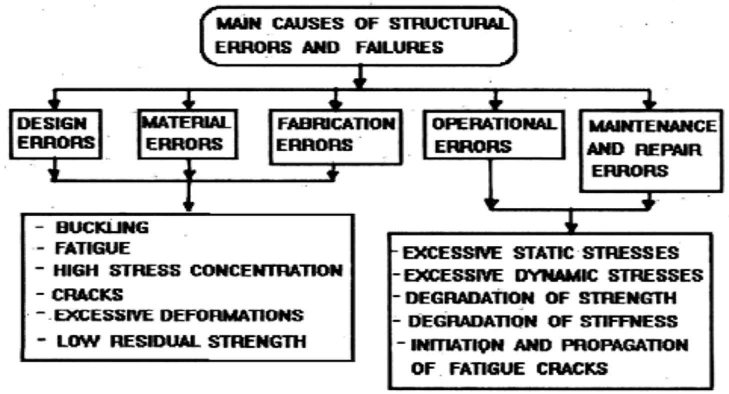


Fig.(54). Main causes of ship structural failure

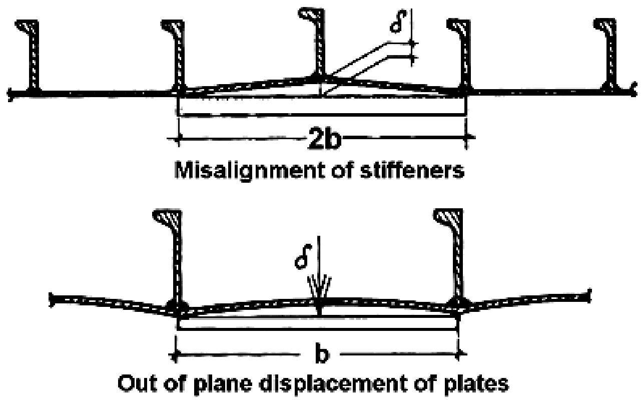


Fig.(55). Misalignment and out of plane distortions

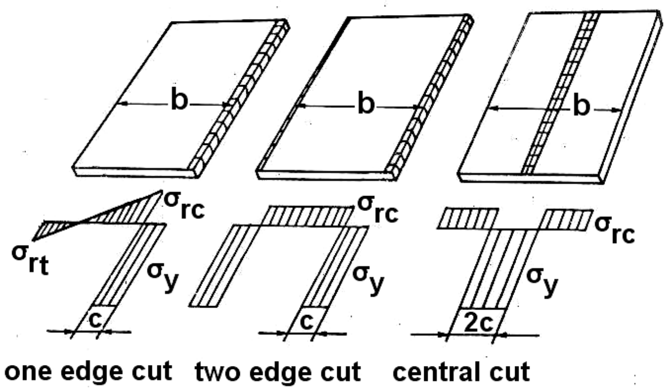


Fig.(56) Residual stresses induced by flame cutting of steel plates

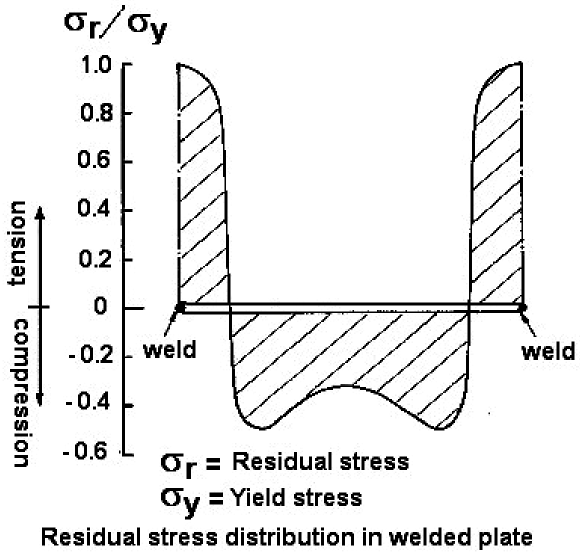


Fig.(57). Residual stress distribution in a welded plate

The failure limit states includes ductile and fracture modes of failure. The ductile mode of failure includes single mode and multiple modes of failure. The single mode of failure includes

elastic and inelastic buckling. The multi modes of failure includes combined elastic buckling and inelastic buckling and plasticity, see Fig.(58).

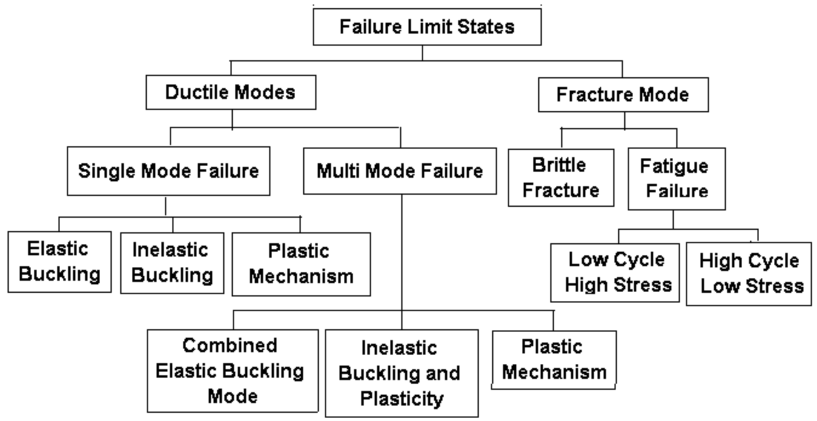


Fig.(58) Failure limit states of ship structures

<u>Categories of ship structural failures</u>

Ship structural failures can be categorized as, see Fig.(59).

- **Hull girder failures**
- **Local structural failures**

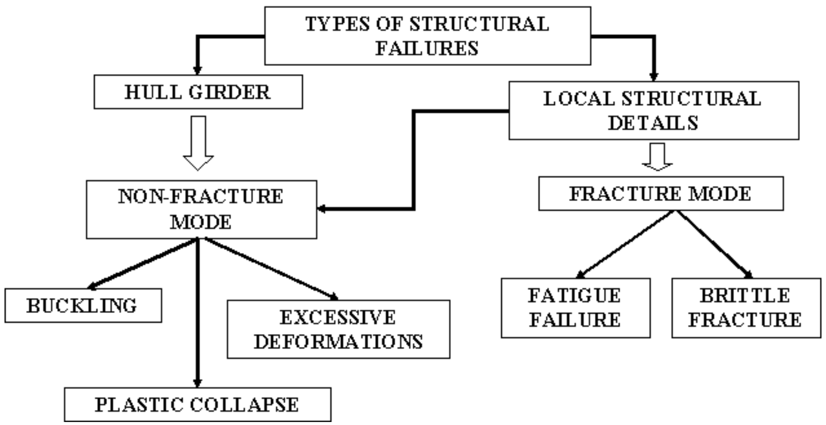


Fig.(59).Different categories of ship structural failures

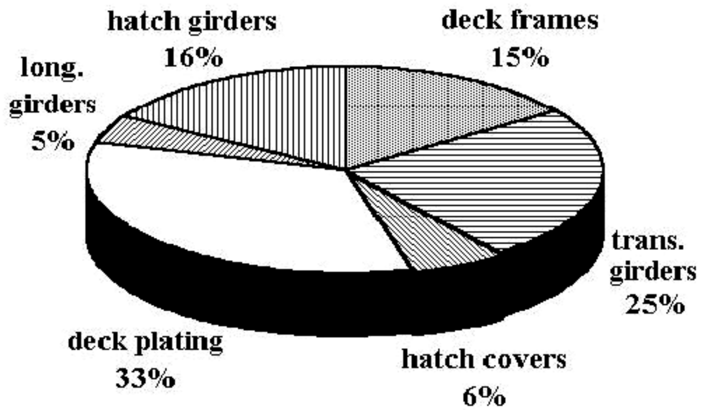


Fig.(60).Buckling failure of deck structural members

These types of failures could be categorized as cracks, fractures, buckling and excessive deformations. The modes of failure are categorized as fracture and non-fracture types, see

Fig. (59). The frequency of buckling failure of the deck structure of general cargo ship is shown in Fig, (60).

The structural failures of brackets in general cargo ships are shown in Fig. (61).

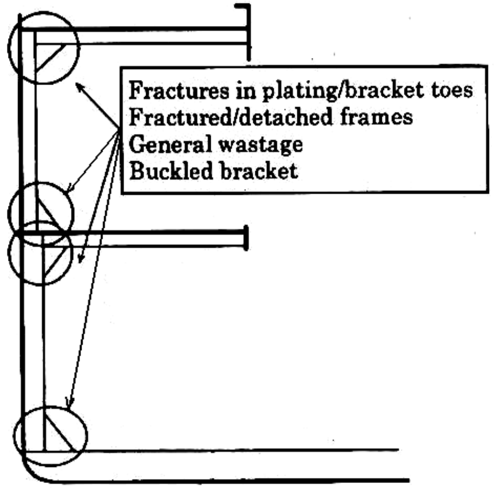


Fig.(61).Structural failures of brackets in general cargo ships

Buckling mode of failure of ship structure (Instability)

Buckling or instability causes excessive lateral deflection. In a strut or columns, lateral deflection occurs. In a plate panel, wrinkling occurs, in a cylinder under radial pressure, corrugation of circumference occurs, in a plate stiffener combination, torsion tripping may occur, etc.

These modes of failure are characterized by a relatively rapid increase in deflection for a small increase in load.

Buckling of ship plates, stiffening members and stiffened panels represent one of the main modes of ship structural failures. The main causes of buckling failure of ship structural elements are shown in Fig.(62).

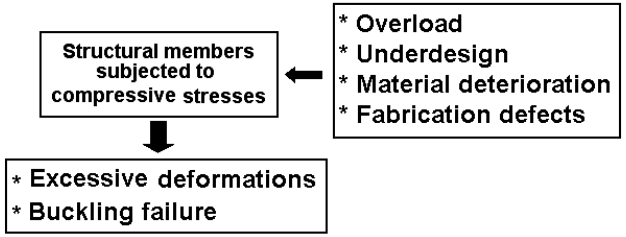


Fig.(62)Causes and consequences of buckling failure

Unacceptable deformations

A structural element having sufficient strength may not necessarily have sufficient stiffness. Large deflections may therefore take place. Although these deflections, or deformations, may not cause structural failure they may have adverse effects on structural performance, particularly under compressive and dynamic loading. Deficient structural stiffness may cause excessive deformations and amplitudes of vibration, see Fig. (63).

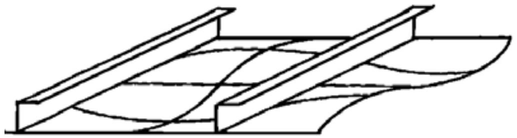


Fig.(63) Excessive deformations

Impact of corrosion on structural strength and stiffness

- **Reduction of local and hull girder scantlings**
- **Reduction of load carrying capacity**
- **In crease of hull girder and local stresses**
- **Reduction of hull girder and local flexural rigidity**
- **Reduction of buckling strength**
- **Reduction of fatigue strength, etc.**
- **The effect of corrosion of the deck plating on the deck section modulus and ultimate strength of the ship is shown in Fig.(64).**

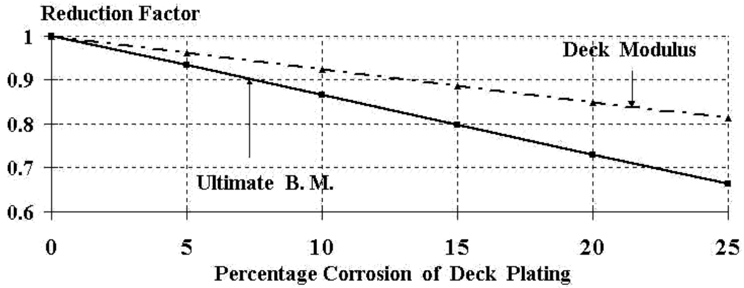


Fig.(64)Effect of corrosion of deck structure on ultimate bending moment

The effect of corrosion on the flexural rigidity of a panel of plating for different plate thicknesses is shown in Fig.(65).

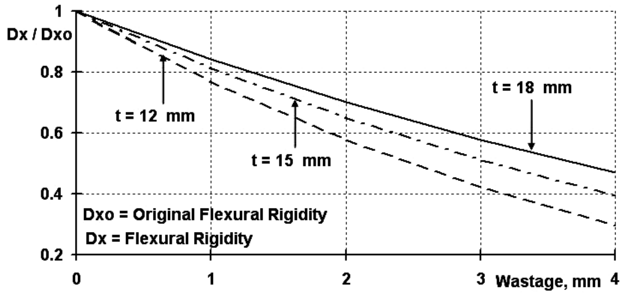


Fig.(65) Effect of plating wastage on flexural rigidity

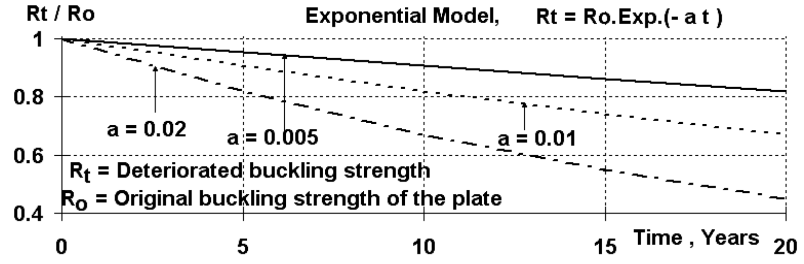


Fig.(66) Variation of plate buckling strength with time (Exponential Model)

The effect of corrosion on the flexural buckling of a plate panel is shown in Fig.(66) for an assumed exponential model of plate deterioration and Fig.(67) for an assumed parabolic model of plate deterioration. Fig. (68) is based on the assumption that the variation of plate thickness with time is given by:

$t_t = t_o(1 - at^2)$

Assuming an exponential model to represent the variation of plate thickness with time, the following model could be used:

$t_t = t_o.\exp.(-bt)$

Where: to =original plate thickness before start of corrosion

t_t = plate thickness at time "t" after corrosion

a and b = factors dependent on the rate of corrosion

Fig. (67) is based on the assumption that the plate thickness variation follows a parabolic model.

The effect of corrosion on the magnitude of the section modulus of frames and longitudinals having different configurations are shown in Fig.(68,69).

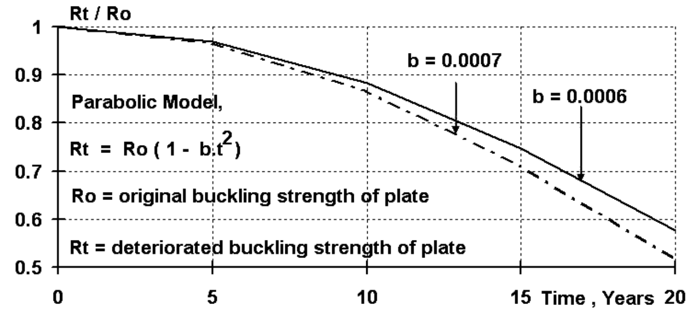


Fig.(67) Variation of plate buckling strength with time (Parabolic model)

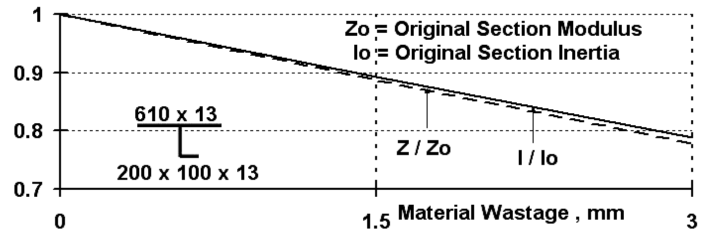


Fig (68) Effect of material wastage on Z/Zo and I/Io of an angle section

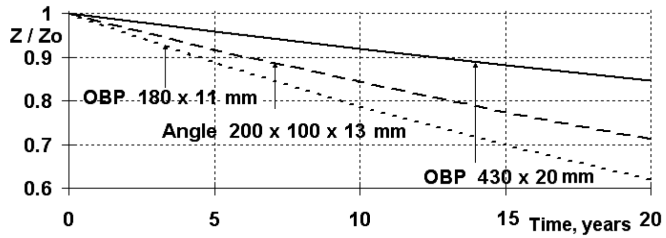


Fig.(69). Variation of Z/Zo of OBP and angle sections with time

Typical examples of buckling failure of ship structure

1- Buckling of the web plating of the transverses in the top wing tanks of a bulk carrier is shown in Fig.(70).

2- Buckling of the floor plating in the double bottom of a general cargo ship is shown in Fig.(71).

3- A typical plate buckling of a stiffened panel, see Fig.(72)

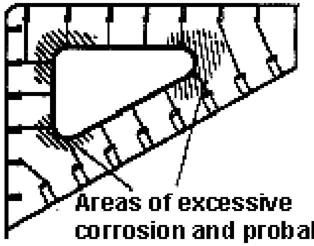


Fig.(70). Buckling of the corroded web plates of top wing tanks

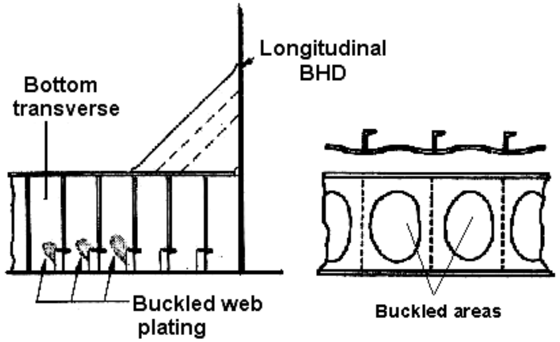
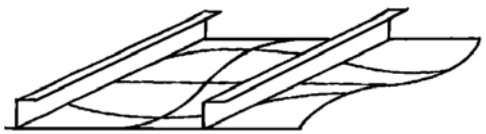


Fig.(71). Buckling of floors and bottom transverses

Fig.(72). Plate buckling of a stiffened panel



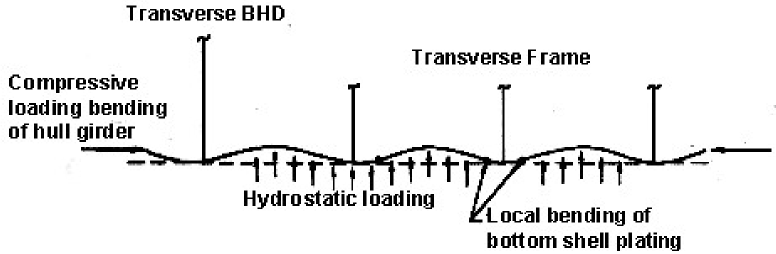


Fig.(73) Mode of buckling of bottom plating of oil tankers

1- **Buckling of bottom plating of an oil tanker is shown in Fig.(73).**
2- **Inelastic buckling of the deck and bottom structure of an oil tanker is shown in Fig.(74).**

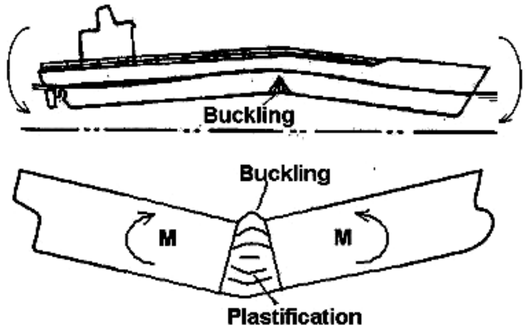


FIG.(74). Plastic collapse under sagging/hogging moments

Control of ship structure failure

Most of ship structural failures result from:

- **Design errors**
- **Material errors**
- **Fabrication errors**
- **Excessive corrosion (lack of proper maintenance).**

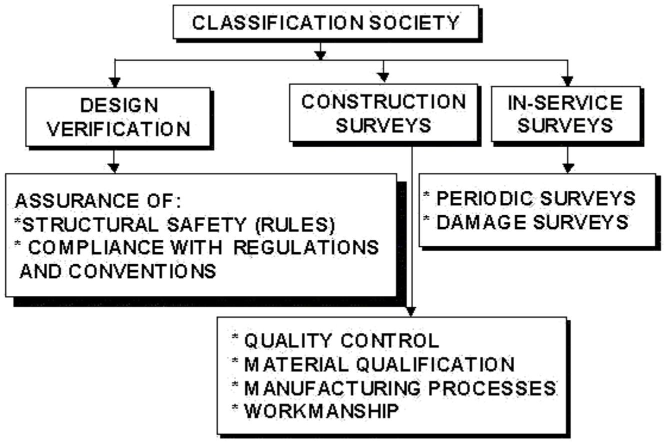


Fig.(75). Role of Classification Societies in controlling ship structure failures

Classification societies play a major role in ensuring adequate safety of ship hull girder and local structural details, see Fig. (75). Ship designers should also pay much attention to the design of ship structural details, as poor design of these structural details is responsible for most of the local structural failures.

Control of buckling of a plate panel subjected to compressive loading could be achieved by fitting a stiffener as shown in Fig. (76).

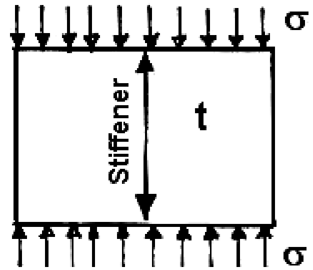


Fig.(76). Improving plate buckling by adding a stiffener

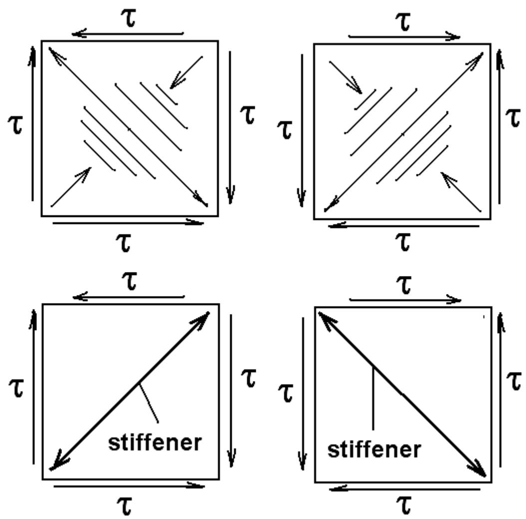


Fig.(77). Control of buckling failure of plate panels under shear loading

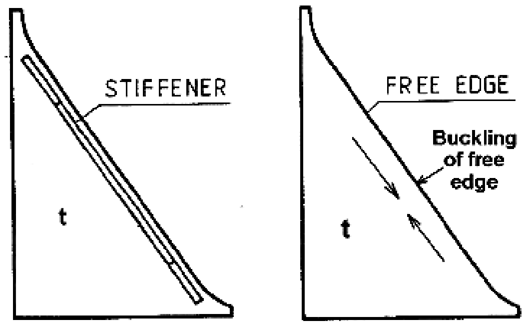


Fig.(78). Buckling control of the free edge of brackets

Plate buckling failure of the transverse webs of top wing tanks of bulk carriers could be repaired by adding stiffeners, see Figs. (79)

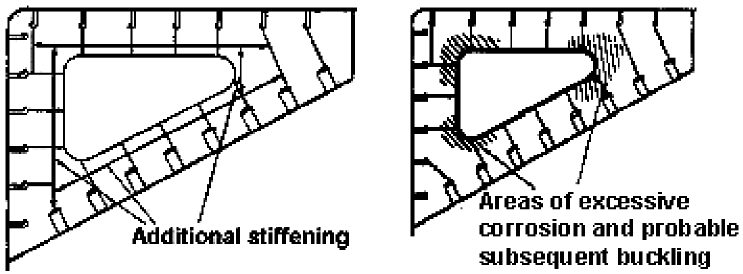


Fig.(79).Buckling of the web frames of the top wing tanks and correction measures

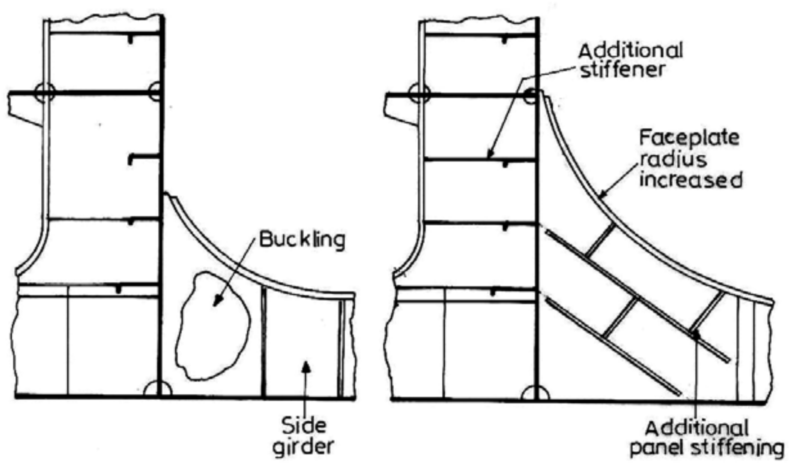


Fig.(80). Additional panel stiffening of the ends of side girders in oil tankers

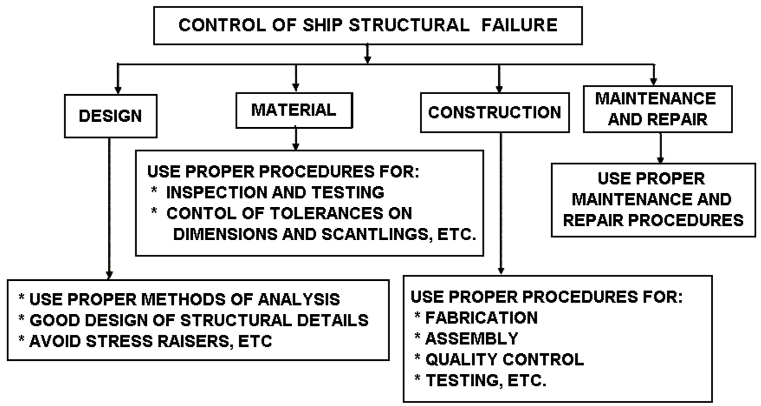


Fig.(81). Control of ship structural failures

Hull girder and local stresses

The loading on ship hull girder and its structural components induce primary, secondary, tertiary and local stresses, see Fig.(82)

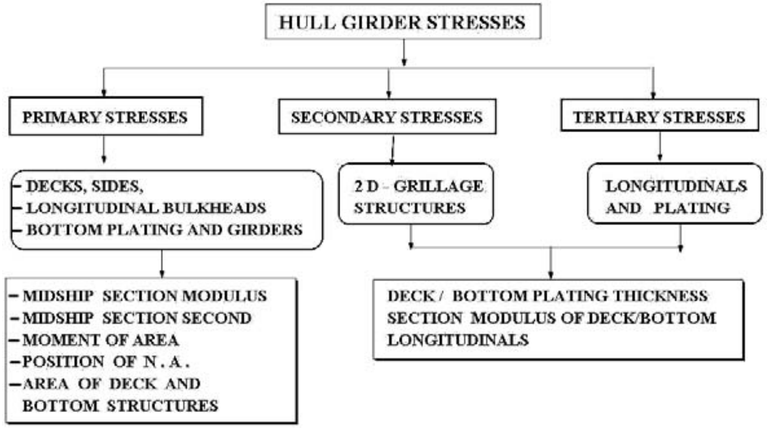


Fig.(82). Hull girder stresses

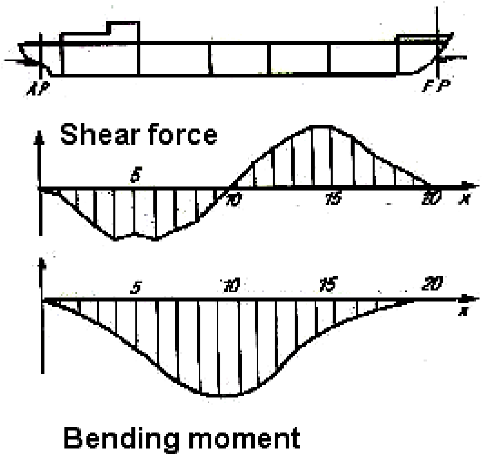


Fig.(83) Hull girder shear force and bending moment

Hull girder loading

A ship hull girder among waves is subjected to:

- **Vertical shear force and bending moment, see Fig.(83).**
- **Horizontal shear force and bending moment**
- **Torsional moment**
- **Local loading**

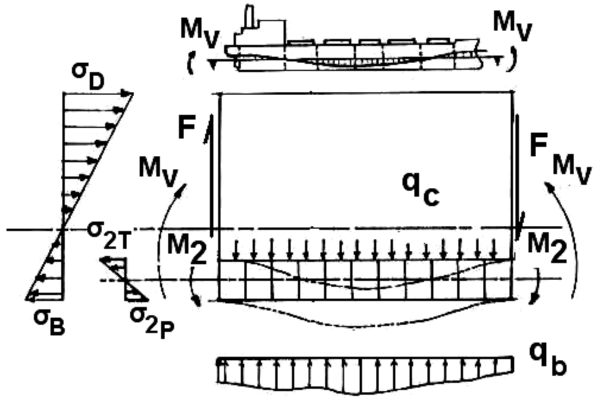


Fig.(84). Hull girder vertical shear force and bending moment

The primary and secondary stresses acting on a ship section and the double bottom structure respectively are shown in Fig. (84).

Hull girder vertical shear force and bending moment

A hull girder in a seaway is subjected to a vertical shear force F_v and bending moment M_v given by, see Fig.(83):

$$M_v = M_s + M_w$$

$$F_v = F_s + F_w$$

Where: F_s and M_s = Stillwater shear force and bending moment

F_w and M_w = Wave shear force and bending moment

Hull girder stresses induced by vertical shear forces and bending moments

The hull girder bending stresses induced by a vertical bending moment in a sagging condition is shown in Fig.(85).

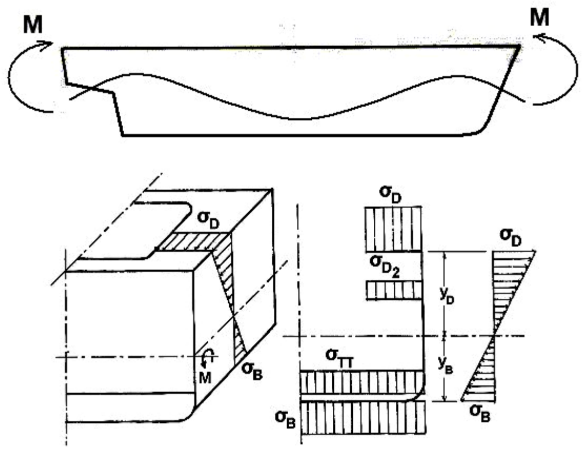


Fig.(85). Hull girder stresses due to vertical bending moment

The hull girder stress at the bottom and deck plating are given by:

$$\sigma_B = \frac{M \cdot y_B}{I} \quad \text{and} \quad = \frac{\cdot}{\quad}$$

Where: M = Total hull girder bending moment

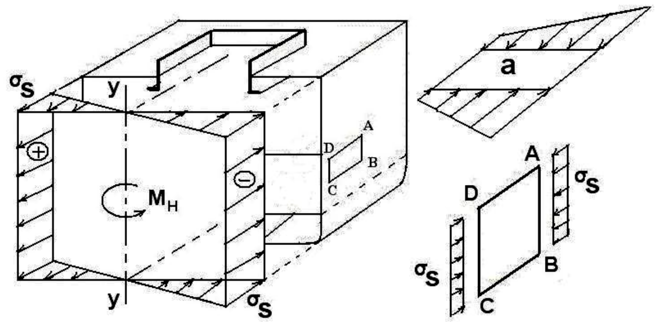
I = Second moment of area of ship section

y_B = Distance of bottom plating from ship section neutral axis

y_D = Distance of deck plating from ship section neutral axis

Hull girder horizontal bending moment

The hull girder of a ship subjected to a horizontal bending moment M_H will induce normal stresses as shown in Fig.(86). The horizontal bending moments induce tensile stresses in either the port side shell and compressive stresses in the starboard side shell or vice versa.



Fg.(86). Hull girder stresses induced by horizontal bending moment

The hull girder stresses induced in the side shell plating is given by:

$$= \frac{\cdot}{\quad}$$

Where:

MH = horizontal bending moment

B = ship breadth

I_y = second moment of area of ship section about the y-axis

Shear stress distribution

The hull girder shear force and bending moment distributions along the length of a bulk carrier is shown in Fig.(87).

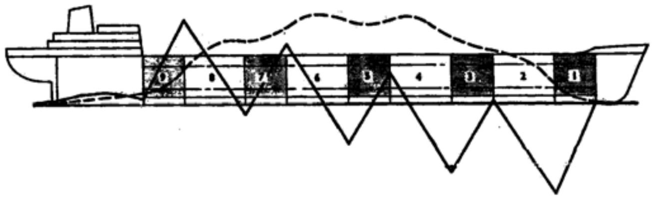


Fig. (87). Hull girder shear force and bending moment

The shear stress distribution over a bulk carrier and tanker ship sections are shown in Fig,(88).

The shear stress is given by: $\tau = \frac{q}{t}$

Where: q = shear flow

The shear flow could be calculated using the general formula given by:

$$q = \frac{F}{I} \sum a\bar{y}$$

Hence, the shear stress is given by: $\tau = \frac{F.\sum a\bar{y}}{I.t}$

Where: F = vertical shear force

I = Second moment of area of ship section

t = thickness of plating

$\sum a\bar{y}$ = Moment of area about ship neutral axis

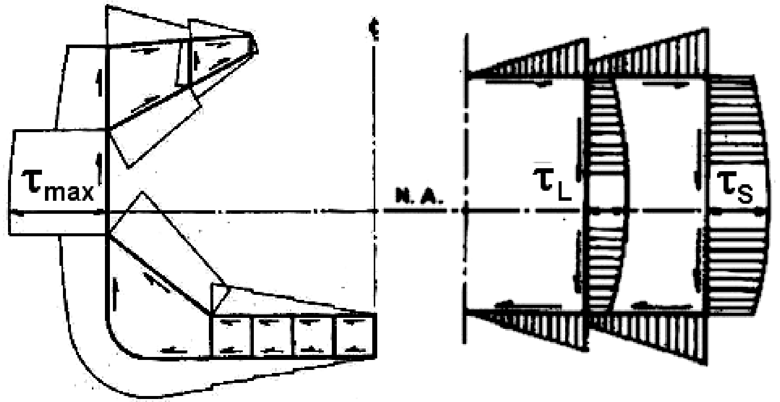


Fig.(88).Shear stress distribution for a bulk carrier and oil tanker

Buckling of ship structure

Buckling of the bottom structure

Buckling of bottom plating

The general loading of bottom plating is as shown in Fig.(89).

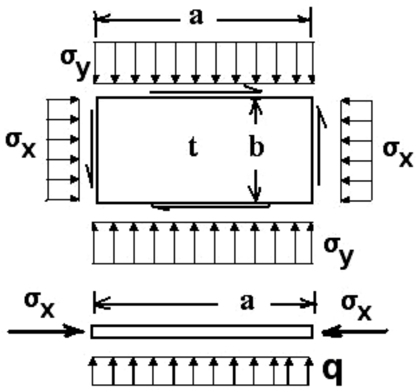


Fig.(89). General loading conditions of ship bottom plating

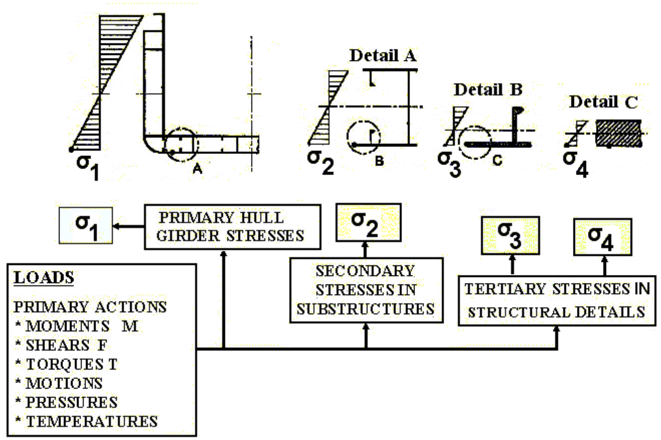


Fig.(90).Hull girder primary and secondary stresses

The various stress components acting on the bottom plating of a ship is show in Fig.(90)

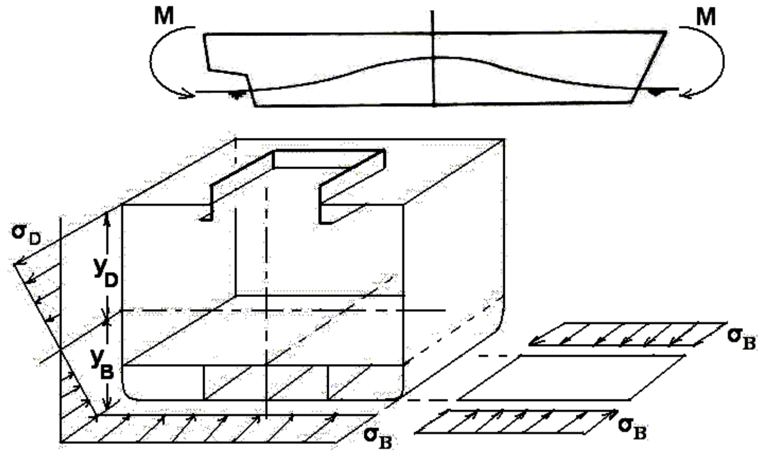


Fig.(91). Hull girder stresses induced in bottom plating

The bottom structure of a ship is subjected to in-plane compressive stresses when the hull girder is subjected to a hogging moment, see Fig.(91).

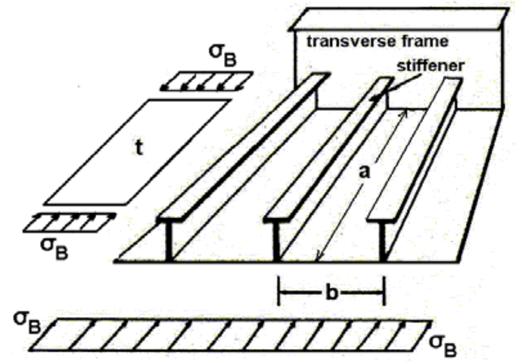


Fig.(92) Hull girder stress in bottom plating

The primary hull girder stress σ_B is given by: $\sigma_B = \dfrac{M \cdot y_B}{I}$

Where: M = hull girder bending moment

y_B = distance of bottom plating from the neutral axis of the ship section

<u>Buckling of bottom longitudinals</u>

The various stress components affecting a bottom longitudinal are, see Fig. (93):

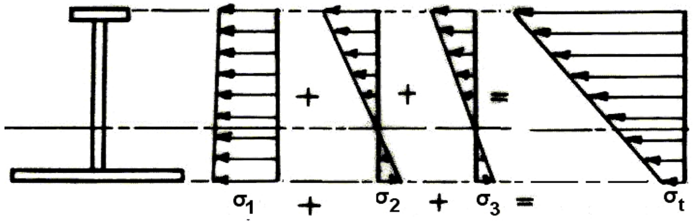


Fig.(93). Stress components affecting bottom longitudinals

σ_1 = Hull girder primary stress

σ_2 = Secondary stress

σ_3 = Tertiary stress

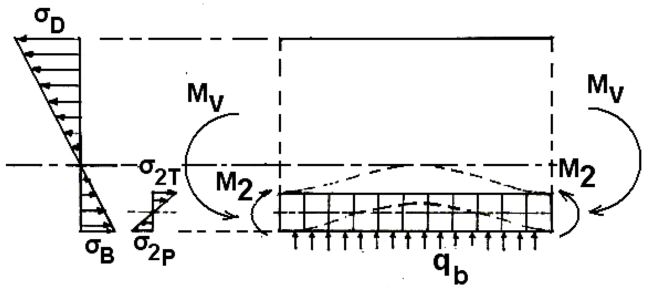


Fig.(94). Hull girder and secondary stresses induced in a double bottom structure

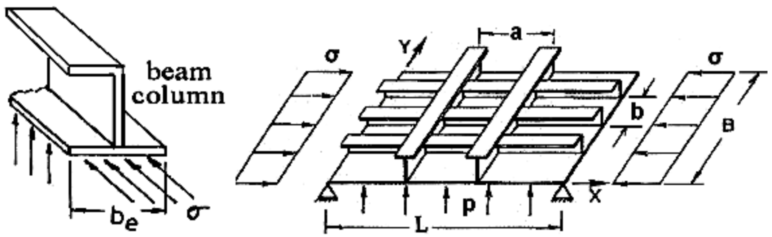


Fig.(95). Secondary loading on a bottom structure

The structural configuration and loading of bottom longitudinals are shown in Fig.(95, 96, 97).

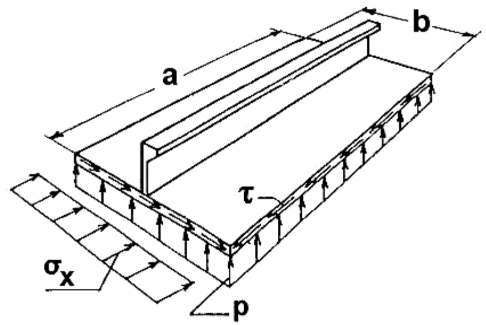


Fig.(96). Local loading on bottom longitudinals

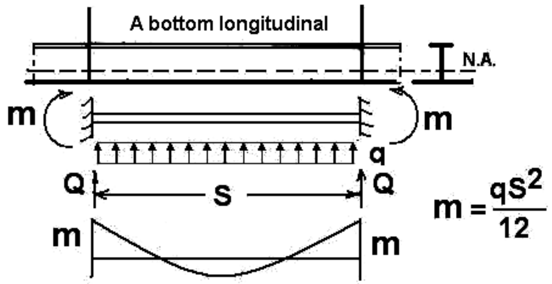


Fig.(97). Secondary stresses in a bottom longitudinal

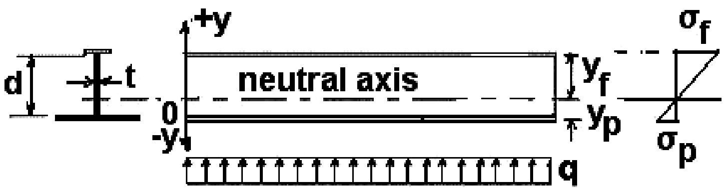


Fig. (98). Tertiary stress in a bottom longitudinal

The tertiary stress σ_3 at the attached plating of the longitudinal is given by:

$$= \frac{\cdot}{\quad}$$

The tertiary stress at the flange of the longitudinal is given by:

$$= \frac{\cdot}{\quad}$$

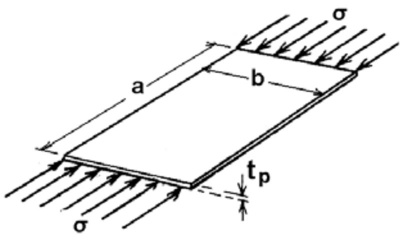
The tertiary stress component due to local loading of bottom longitudinals is shown in Fig.(98).

<u>Buckling of bottom plating</u>

It is assumed that the plate panels are simply supported.

The Euler buckling stress is given by:

$$\sigma_e = \frac{k \cdot \pi^2 \cdot E}{12(1-\upsilon^2)} \left(\frac{t}{b}\right)^2$$



Where: σ_e = Euler buckling stress

ν = Poisson's ratio, ν = 0.3

E = Modulus of elasticity, (= ×)

K = Constant depending on a/b ratio

When / >> , =

t = plate thickness

b= length of short side of plate

Elastic buckling occurs when: ≤

Where: σ_y = yield stress of the plate material

Inelastic buckling occurs when: $\sigma_e \geq 0.5\, \sigma_y$

In this case, the critical buckling stress is given by:

$$\sigma_{cr} = \sigma_y \left(1 - \frac{\sigma_y}{4\sigma_e}\right)$$

<u>Total stress affecting bottom plating</u>

The various stress components affecting the bottom plating of a ship is shown in fig. (99)

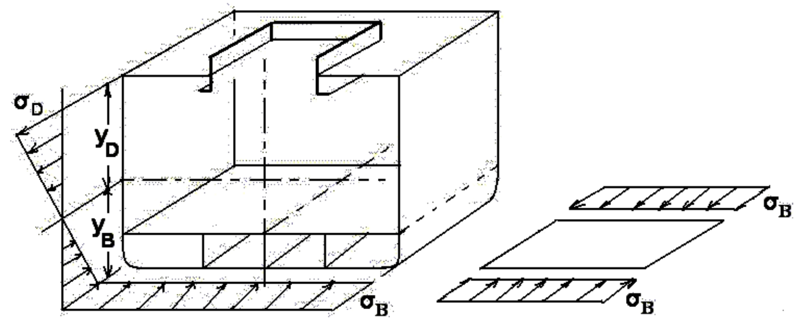


Fig. (99). Various stress components affecting bottom plating

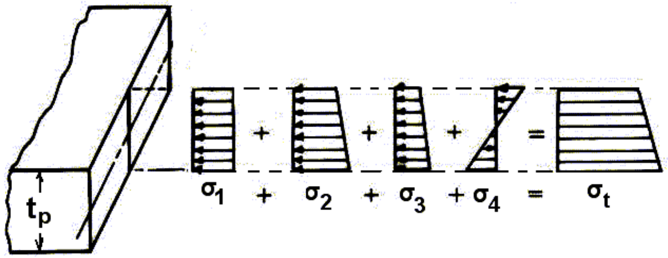
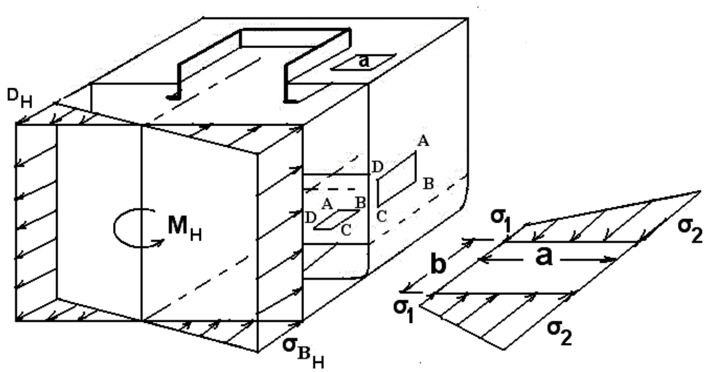


Fig.(100).Hull girder stresses induced by vertical hogging moments

The total in-plane stress is given by: $\sigma = \sigma_B + \sigma_{local}$

Where: σ_B **= hull girder stress**



σ_{local} **= local stress**

Fig. (101). Hull girder stresses induced by horizontal bending

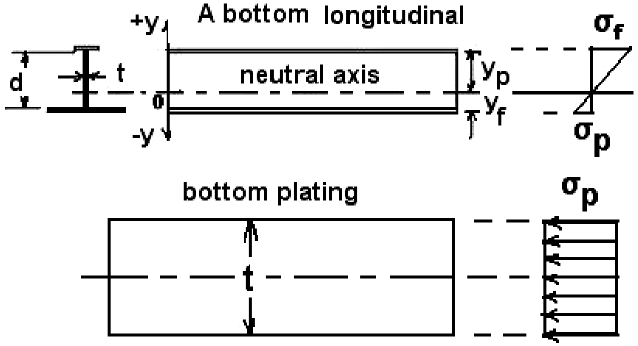


Fig. (102).Secondary stresses induced by bending of bottom longitudinals

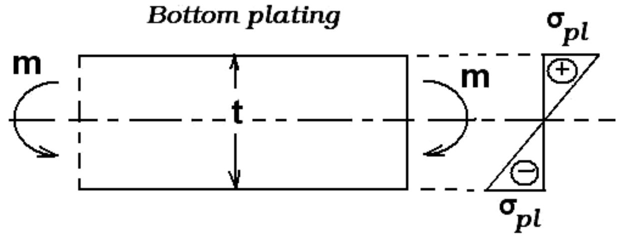


Fig.(103).Tertiary stressing bottom plating

<u>Tertiary stresses induced by local loading</u>

The local stresses due to plate bending is given by, see Fig (103)

$$_{pl} = \frac{.}{\quad} = \frac{.}{3} = \frac{}{2}$$

Where: m = $qL^2/12$

t = plate thickness

Buckling of deck structure of a ship

A deck structure of a general cargo ship could be either transversely or longitudinally framed). The deck structure is subjected to in-plane tensile or compressive stresses, shear stresses and bending stresses. The in-plane normal and shear stresses are induced by the hull girder bending moments. A ty[ical arrangement of a transversely framed deck structure is shown in Fig.(104).

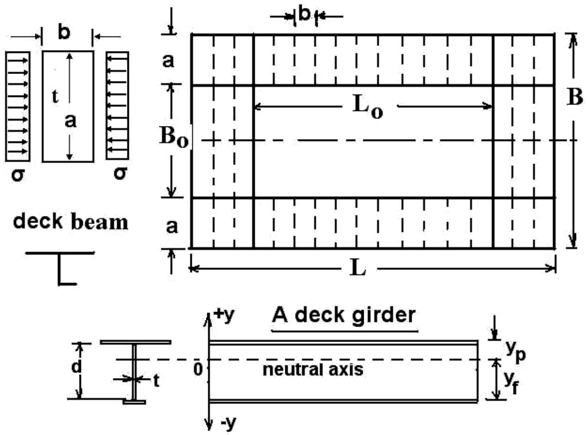


Fig.(104). Transversely framed deck structure

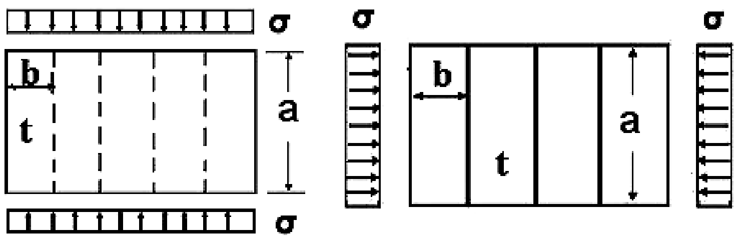


Fig.(105). General loading of the deck structure of a ship

General loading of deck structure

The main load components affecting a deck structure of a ship are:

- **Stresses induced by hull girder bending**
- **Secondary loading on stiffened panel of the deck structure**
- **Local loading on deck longitudinals between deck transverses**
- **Local loading on deck plating**

The local loading of the deck structure of a ship is shown in Fig.(106).

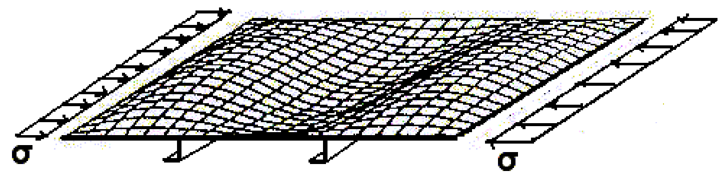


Fig.(106). Local loading of the deck structure

Buckling of deck structure of a ship should cover:

1. **Buckling of deck longitudinals**
2. **Buckling of deck plating**
3. **Buckling of deck girders**

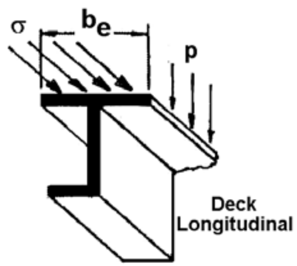


Fig. (107). Local loading on a deck longitudinal

Deck stresses induced by hull girder bending

When the hull girder of a ship is subjected to a sagging bending moment, see fig.(108), the deck structure will be subjected to compressive stresses.

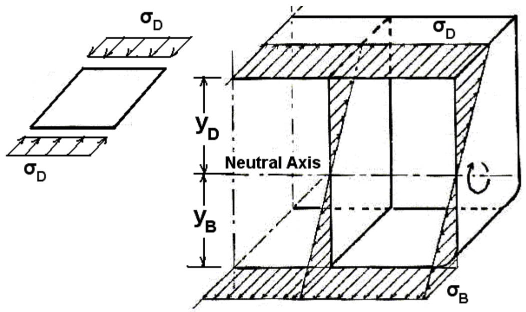


Fig. (108). Hull girder bending stresses in a sagging condition

The primary hull girder stress at the deck of a ship section is given by, see fig. (109): $\sigma_D = \dfrac{M \cdot y_D}{I}$,

Where: M = hull girder bending moment

I = the second moment of area of ship section about the neutral axis of the ship section

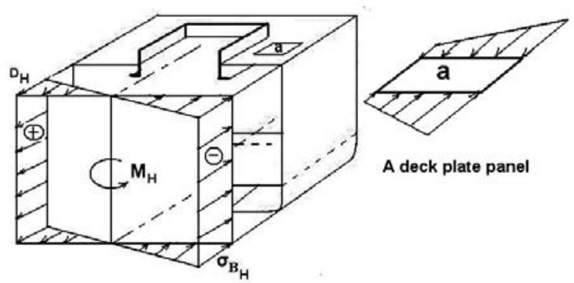


Fig.(109).Deck stresses induced by horizontal bending moments

Buckling of deck longitudinals

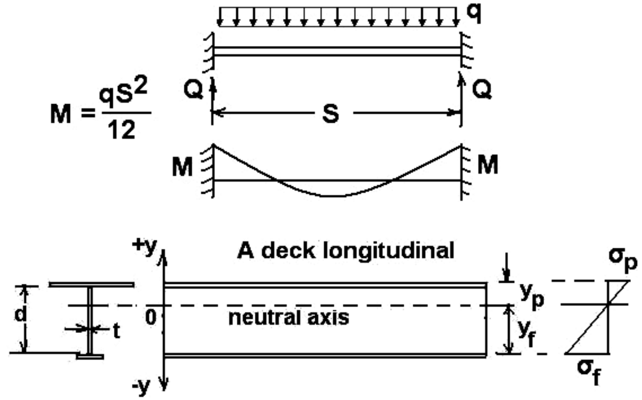


Fig. (110). Local bending stress of a deck longitudinal

A deck longitudinal is subjected to the primary, secondary and tertiary stresses.

The local bending stress in a deck longitudinal induced by the local deck loading is given by, see Fig. (110):

$$\sigma_{locp} = \frac{m \cdot y_P}{i}$$

Where:

m = bending moment affecting the local structural element

i = second moment of area of the section of member

y_f = distance of the flange of section from the neutral axis of the section

The various stress components affecting a deck longitudinal is shown in fig.(111).

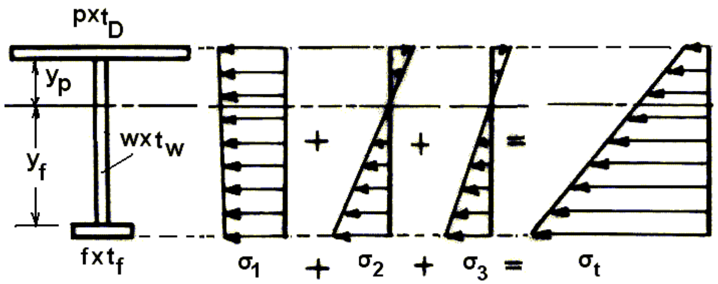


Fig.(111).Various stress components of a deck longitudinal

Buckling of deck girders

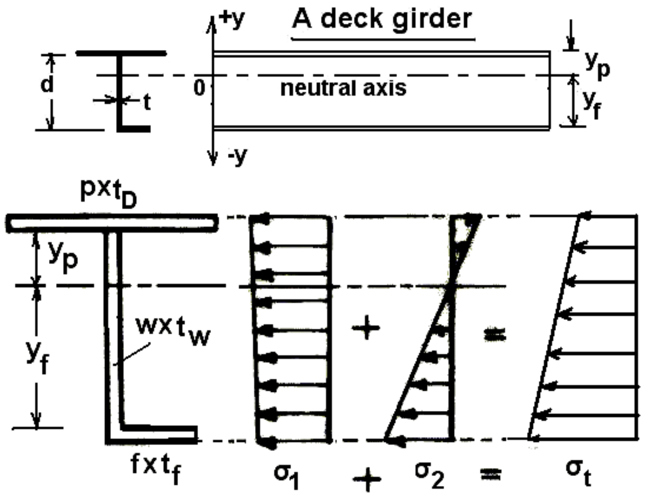


Fig. (112).Primary and secondary stresses in a deck girder

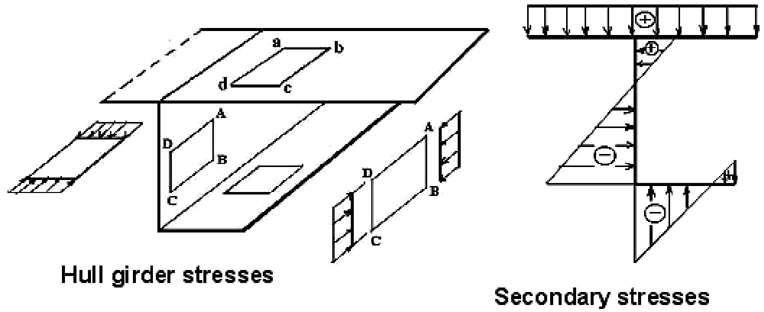
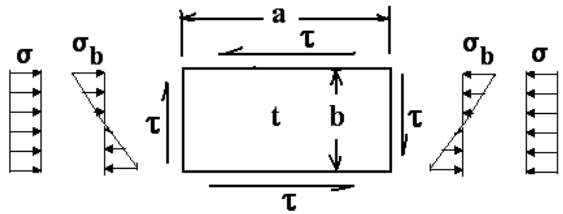


Fig. (113). Hull girder and secondary stresses in deck longitudinals

The flange and web plate of the deck girder are subjected to uniform compressive stresses induced by hull girder sagging bending moments and linear stress distribution induced by secondary and local loading, see Fig.(113).

Therefore, the web and flange panels of plating of deck girders are subjected to a combined system of stresses. The buckling strength of these panels could be evaluated using the following equation:

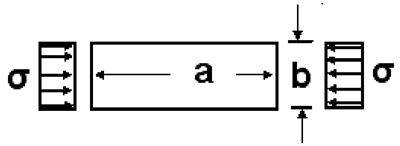
$$\frac{\sigma}{\sigma_{ec}} + \left(\frac{\sigma_b}{\sigma_{eb}}\right)^2 + \left(\frac{\tau}{\tau_{es}}\right)^2 \leq 1.0$$



Where σ_e = Euler buckling stress

The Euler buckling stress, in this case is given by:

$$\sigma_e = \frac{\pi^2 E}{12\left(1-\upsilon^2\right)}\left(\frac{t}{b}\right)^2 \cdot k$$



Buckling of deck plating

Buckling of fixed and simply supported transversely ramed deck plate is shown in Fig. (114).

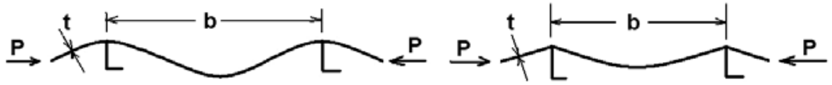


Fig.(114). The buckled shape of a deck plate assuming fixed and simply supported ends

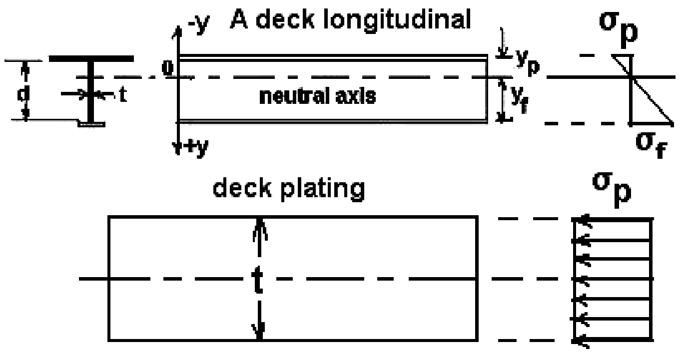


Fig.(115).Stresses induced in deck plating by local bending of deck longitudinals

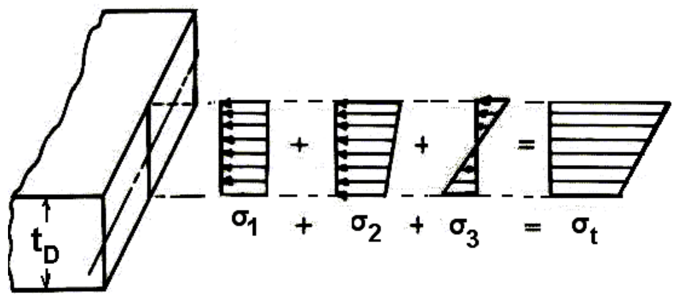


Fig.(116). Various stress components affecting deck plating

The various stress components affecting the deck plating of a ship is shown in fig. (116). The tertiary stress in the deck plating is given by, see Fig. (115):

$$= \frac{\cdot}{\quad}$$

Where: m = local bending moment on bottom longitudinal
$m = qL^2/12$
q = transverse loading on deck longitudinal
L = span pf longitudinal
i = second moment of area of section of the deck longitudinal
y_p = distance of deck plating from neutral axis of the longitudinal section

Plate buckling occurs when:

- When: $\sigma_e \le \sigma_y/2$ **then:** $\sigma_{cr} = \sigma_e = \frac{k\pi^2 E}{12\left(1-\upsilon^2\right)}\left(\frac{t}{s}\right)^2$

- When: $\sigma_e > \sigma_y/2$ **then:** $\sigma_{cr} = \sigma_y\left[1-\frac{\sigma_y}{4\sigma_e}\right]$

σ_{cr} = critical buckling stress

Buckling of side shell structure

The hull girder bending stress over the side shell structure of a general cargo ship is shown in Fig.(117).
The primary hull girder stresses over a panel of plating "abcd", see fig.(94), is given by: $\sigma_1 = \frac{M.y_1}{I}$ and $\sigma_2 = \frac{M.y_2}{I}$

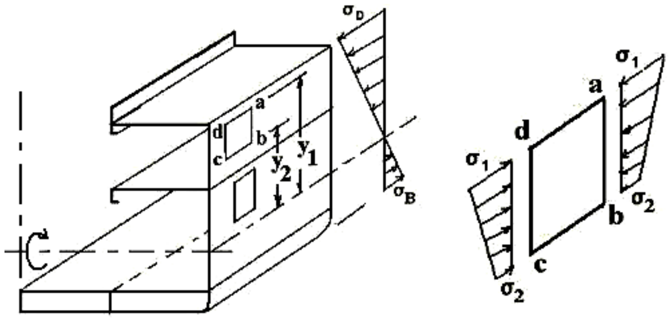


Fig. (117).Side shell stresses due to vertical bending moment

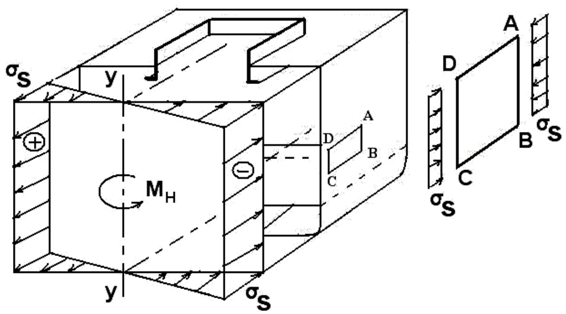


Fig. (118). Side shell stresses due to horizontal bending moments

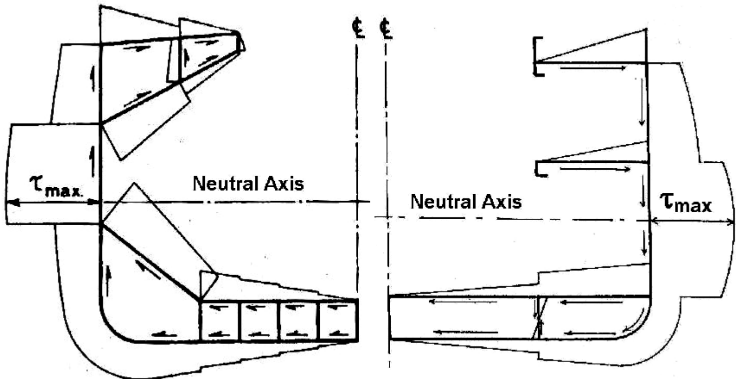


Fig. (119). Shear stress distribution over ship sections of a general cargo ship and a bulk carrier

The shear and bending stresses induced in a side shell plate at the ship section neutral axis is shown in Fig.(120) and for a panel above the ship neutral axis is shown in Fig.(121) and for a panel below the neutral axis is shown in Fig.(121).

Total stresses in the side shell plating

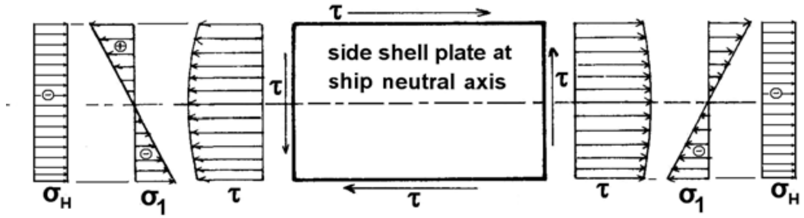


Fig.(120).Shear and bending stresses in a side shell plate at the ship section neutral axis

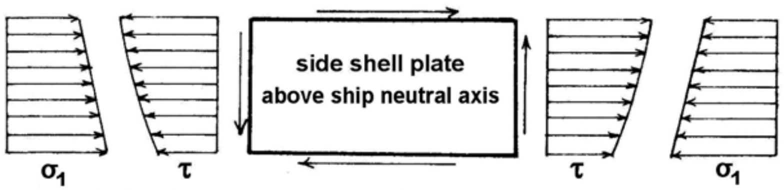


Fig.(121).Shear and bending stresses in a side shell plate below ship section neutral axis

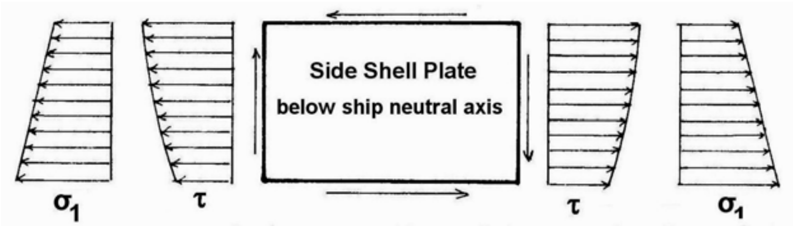


Fig.(122).Shear and bending stresses in a side shell plate above ship section neutral axis

A plate panel of the side shell is therefore subjected to a combined system of stresses as shown in fig.(123), as the side shell is also subjected to shear stresses.

The equation governing critical buckling is given by:

$$\frac{\sigma}{\sigma_{ec}} + \left(\frac{\sigma_b}{\sigma_{eb}}\right)^2 + \left(\frac{\tau}{\tau_{es}}\right)^2 \leq 1.0$$

Where: $\sigma = \frac{\sigma_1 + \sigma_2}{2}$ **and** $\sigma_b = \frac{\sigma_1 - \sigma_2}{2}$

$$\sigma_{ec} = \frac{\pi^2 E}{12(1-\upsilon^2)}\left(\frac{t}{b}\right)^2 \cdot k_c$$

$$\sigma_{eb} = \frac{\pi^2 E}{12(1-\upsilon^2)}\left(\frac{t}{b}\right)^2 \cdot k_b$$

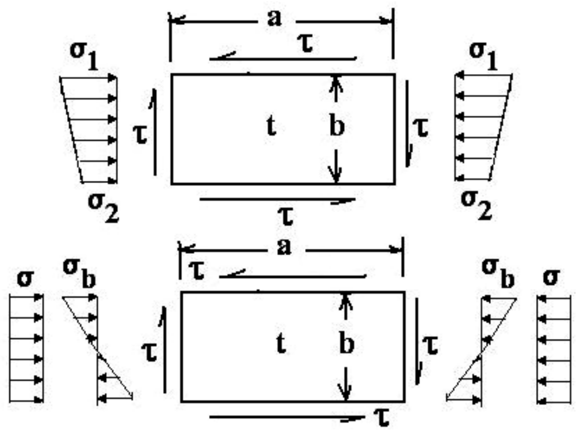


Fig. (123).. Plate panel subjected to in-plane, bending and shear stresses

Ultimate strength of ship plating

For a ship structural element, the variation of the bending moment with lateral deflection is as shown in Fig.(124). The collapse behavior of a steel plate under compressive stresses is shown in Fig.(125).

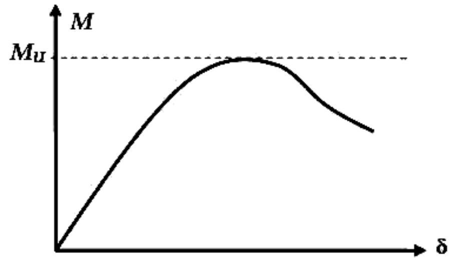


Fig.(124). Bending moment deflection curve

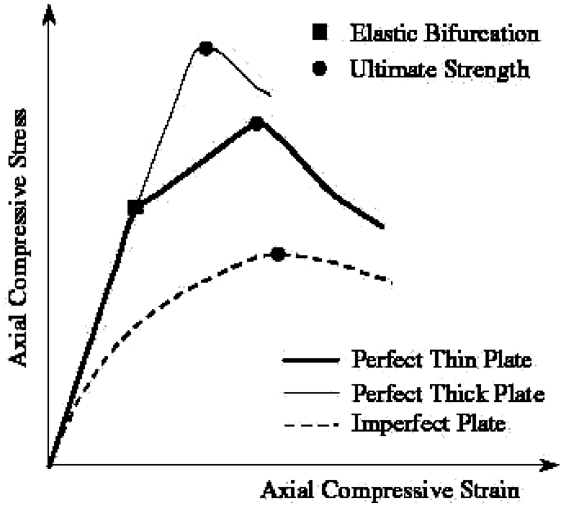


Fig.(125). Collapse behavior of a steel plate under compressive stresses

The collapse of a ship hull girder under a sagging bending moment exceeding the ultimate carrying capacity of the ship section is shown in Fig.(126).

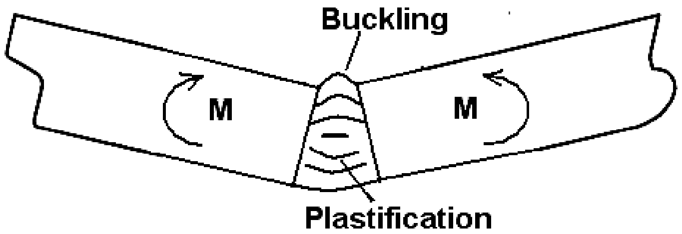


Fig.(126). Collapse of a ship hull girder

Effective width of plating

Because of the shear lag effect, the normal stresses on the plate of a stiffened panel will not be uniform, see Fig.(127).

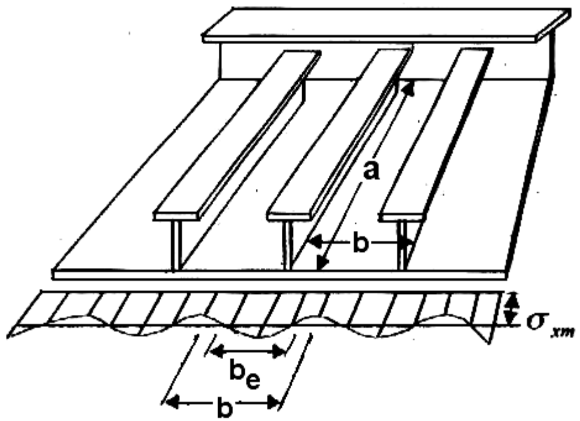
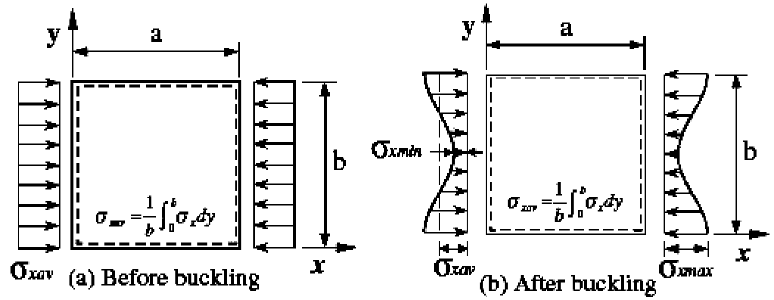


Fig.(127).Shear lag effect in plating of a longitudinally framed bottom structure

The stress distribution over the edge of the plate panel before and after buckling occurs is shown in Fig.(128).



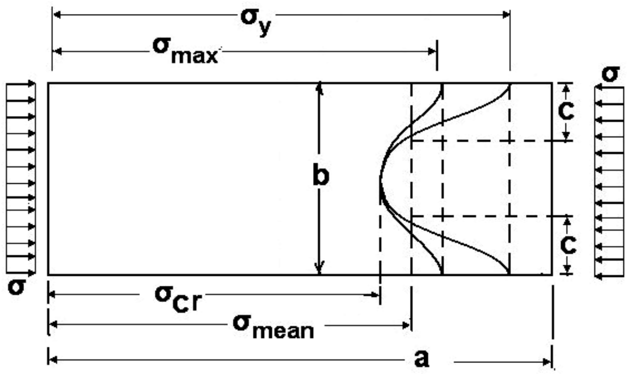


Fig.(128). Variation of the shape of stress distribution

Assume a simply supported rectangular plate loaded with a uniformly distributed load along the short edge of the plate. When the applied load exceeds the critical buckling load, the additional load will be concentrated towards the edges of the plate and the stresses will increase in those regions, see Fig.(128). In Fig.(128) we have:

σ_{cr} = critical buckling stress of plate loaded along the short edges.

σ_{max} = maximum stress along the side-edges

σ_{mean} = mean stress in plating

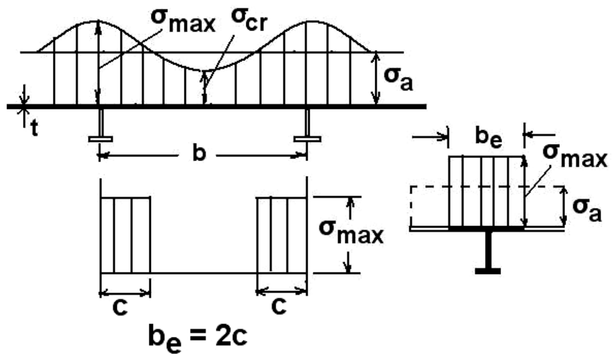


Fig.(129). The effective breadth of stiffeners

The effective width of plating is defined as the width of plating which, when taking the full load, has a uniformly distributed stress equal to σ_{max}. This effective width is represented by a width C from each side edge, see Fig.(129).
Hence the effective width b_e = 2C.
Then the middle portion of the plate can be disregarded and the two strips can be handled as a long simply supported rectangular plate of width 2C.
The critical stress for such a plate is given by:

$$= \frac{}{(- \upsilon)}\left(\frac{}{}\right)$$

Assuming that the ultimate load is reached when σ_{cr} becomes equal to the yield stress σ_y of the material we obtain:

$$= \frac{}{\sqrt{(- \upsilon)}}\sqrt{\frac{E}{\sigma_y}}$$

Hence, the ultimate load is given by:

$$P_u = 2Ct\sigma_y = \frac{\quad}{\sqrt{(\quad - \upsilon\quad)}}\sqrt{E\sigma_y}$$

Where: $\nu = 0.3$, hence: $P_u = 1.9t^2.\sqrt{E\sigma_y}$

An approximate expression for "C" is given by:

$$C = 1.9(1 - 0.55\sqrt{\frac{E}{\sigma_y}}\,.\,t/b)$$

Hence: $P_u = t^2\sqrt{E\sigma_y}$ x1.9(1 - 0.55.$\sqrt{E/\sigma_y}$.(t/b))

$$= 1.9\, t^2.E(\sqrt{E/\sigma_y} - 0.55(t/b))$$

The ultimate stress is given by:

$$\sigma_u = \frac{\quad}{\quad} = 1.9.E\frac{\quad}{\quad}(\sqrt{E/\sigma_y} - 0.55(t/b))$$

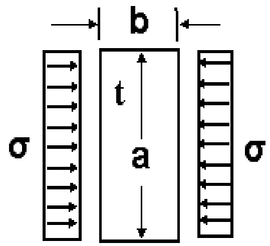
The post–buckling reserve of strength varies with the geometry of the plate, the type of loading and the boundary support conditions.

When $\sigma_e \geq \sigma_y$ or when $b/a < 1/4$ or when one edge is free, there is virtually no reserve of strength, b = length of short edge of plate.

When: $b/a \geq 2.0$, there is large post-buckling reserve of strength.

<u>Ultimate stress of simply supported panels</u>

<u>Case 1- Long edges loaded</u>



The ultimate load is given by:

$P_u = 1.77x\ t^2.\sqrt{E\sigma_y}\ (a/b)^{0.22}$

The ultimate stress is given by:

$$\sigma_u = \frac{\quad}{\quad} = 1.77.\sqrt{E\sigma_y}\ (a/b)^{0.22}\ .\ \frac{\quad}{\quad}$$

$$= 1.77\sqrt{E\sigma_y}\ .\quad ^{0.78}\ .t/bx$$

The ultimate strength is given by:

$$\frac{\sigma_\mu}{\sigma_y} = \frac{1.77\ \alpha}{\beta} \qquad \text{When } \alpha \leq 3.3$$

$$\sigma_\mu = \frac{\pi^2 E}{12(1-\upsilon^2)}\left(\frac{t}{b}\right)^2 \qquad \text{When } \alpha > 3.3$$

Where: $\quad = \frac{\quad}{\quad} =$ Aspect ratio of the plate

The critical buckling stress of the column is then taken as the ultimate stress of plating.

$$\sigma_u = \frac{\pi}{Ab^2(\quad \upsilon^2)}$$

Where: $I = t^3/12$, $A = 1xt$

$$\text{Hence} \quad \sigma_u = \frac{\pi}{12(\quad \upsilon^2)}\left(\frac{t}{b}\right)^2$$

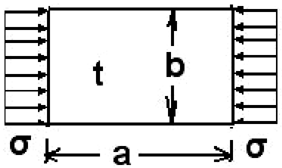
The critical buckling stress of a simply supported panel with long edges in compression is given by:

$$\sigma_e = \frac{\pi}{12(\quad \upsilon^2)}\left(\frac{t}{b}\right)^2\left(1+\left(\frac{b}{a}\right)^2\right)^2$$

When a >>b $[b/a]^2 \to 0$

Hence $\sigma_e = \dfrac{\pi}{12(\quad \upsilon^2)}\left(\dfrac{t}{b}\right)^2 = \sigma_u$

<u>**Case 2: Short edges loaded**</u>



The ultimate strength is given by:

$$\frac{\sigma_\mu}{\sigma_y} = \frac{1.9}{\beta} - \frac{0.8}{\beta^2}$$

Where: σ_u = ultimate failure stress

σ_y = yield stress

β = slenderness parameter

The slenderness parameter β is given by:

$$= -\sqrt{-}$$

a = long dimension of plate

b = short dimension of plate

Alternatively, the effective width could be obtained from the following equation, see Fig.(130):

— = — – —

The ultimate strength for uniaxial compression in short edge was developed by Faulkner, which is written by:

$\sigma_{Ux} = \sigma_0 C_x$ and

$$C_x = \begin{cases} 1.0 & for \quad \beta < 1 \\ \dfrac{2}{\beta} - \dfrac{1}{\beta^2} & for \quad \beta \ge 1 \end{cases}$$

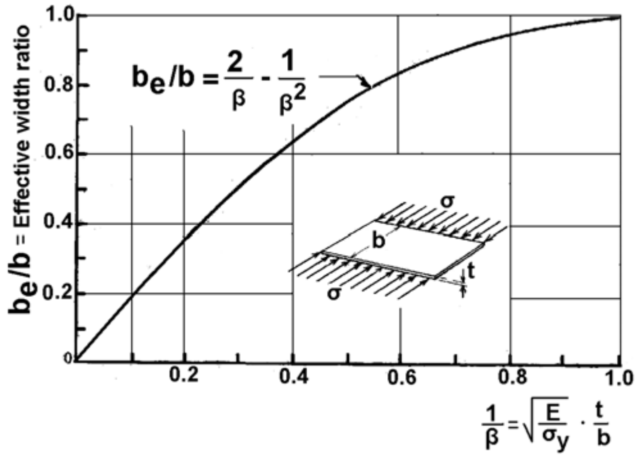


Fig.(118) Effective width of plating

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